Journal of Basrah Researches (Sciences) **48**(2), 44 (2022)
DOI: https://doi.org/10.56714/bjrs.48.2.5

Structure of low-lying states in $^{158\text{-}168}$Hf nuclei

Falih H. Al-Khudair*, Hussein N. Qasim

Department of Physics, College of Education for Pure Sciences, University of Basrah, Basrah, Iraq.

A R T I C L E I N F O

Received 12 June 2022
Accepted 04 September 2022
Published 30 December 2022

K e y w o r d s :

Mixed-symmetry states, Interacting boson model, Mixing ratio, Hf-isotopes.

Citation: F.H. Al-Khudair, H.N. Qasim, J. Basrah Res. (Sci.) **48**(2), 44 (2022). DOI:https://doi.org/10.56714/bjrs.48.2.5

A B S T R A C T

The characteristics of low-lying mixed-symmetry states in $^{158\text{-}168}$Hf isotopes have been investigated in the framework of the interacting boson model. The obtained results of the low energy spectra, B(E2) and mixing ratio δ(E2/M1) for all isotopes are compared with empirical values . The effect of the Majrona terms on the energy levels have been investigated. The 2_{ms}^{+} and 1^{+} states are the lowest mixed-symmetry for vibrational $^{158\text{-}164}$Hf nuclei and for rotational ^{166,168}Hf nuclei, respectively.

1. Introduction

Theoretically, studies of nuclear properties by applying the boson model versions have been extended to all types of nuclei [1-6]. The excitations dealt with symmetry in the properties of energy levels [7-9]. Experimentally, collective low-lying 1^{+} states have been observed in several deformed rare-earth nuclei [10-12]. The presence of 2^{+} states around 2 MeV has been identified as mixed symmetry states in the vibrational nuclei [12-14]. Hafnium nuclei's nuclear structure has previously been studied both theoretically and empirically. K. S. Krane [15] proposed the multipole mixing ratios δ(E2/M1) of gamma transitions in even-even deformed nuclei and obtained reduced E2/M1 mixing ratios of gamma transition from levels in β- and γ- bands to levels in ground states band for ^{152}Sm and ^{178}Hf isotopes. E. Stuchbery [16] examined the overall behavior of g-factors. The IBM-2 calculations have been performed including the g_π and g_ν –factors ratio. Further, he found that small values of B(M1) between low-lying collective states. D .L. Zhang [17] studied the features of the triaxial superdeformed bands levels using a supersymmetry scheme that included many-body interactions. Experimentally, B. Bochev *et al.* [18] measured the lifetimes and B(E2) values in even-even 166-170Hf nuclei using the recoil-distance Doppler-shift method. Using the delayed coincidences with LaBr3(Ce) detectors and an Orange conversion-electron spectrometer. Rudigier et al. [19] have

*Corresponding author email : falih9@gmail.com

ISSN: 1817-2695 (Print); 2411-524X (Online)
Online at: https://bjrs.uobasrah.edu.iq

investigated the energy scheme and yielded new B(E2) values for deformed Hf- isotopes. McCutchan et al. [20] measured lifetimes of excited states in ^{162}Yb and ^{166}Hf using the recoil distance Doppler shift (RDDS) method in coincidence mode. Excitation energy and electromagnetic transition data have been compared to boson model productions. Recently, X. Q. Yang et al [21] have considered five-dimensional Hamiltonian based on covariant density-functional theory to investigate the shape coexistence and rigid triaxial deformation in the Er- Pt isotopic chains (102≤N≤124). Y. B. Choi et al. [22] used the deformed relativistic method to do a systematic examination of shape coexistence in Hf, W, Os, Pt, and Hg nuclei.

2-The interacting boson model

The proton-neutron interacting boson model describes collective low-lying states that are not totally symmetric in the neutron-proton degree of freedom. The general form Hamiltonian can be written as [23]

$$H = \varepsilon_d\left(\hat{n}_{d\nu} + \hat{n}_{d\pi}\right) + \kappa_{\pi\nu}(\hat{Q}_\nu.\hat{Q}_\pi) + \hat{V}_{\nu\nu} + \hat{V}_{\pi\pi} + \hat{M}_{\nu\pi} \quad (1)$$

where ε_d is d-boson energy, $n_{d\pi}$ and $n_{d\nu}$ are the number operator of proton and neutron d-bosons, the second term $\kappa_{\pi\nu}(\hat{Q}_\nu.\hat{Q}_\pi)$ is the quadrupole interaction. The quadrupole operator is given by:

$$\hat{Q}_\rho = (d^\dagger_\rho s_\rho + s^\dagger_\rho \tilde{d}_\rho)^{(2)} + \chi_\rho(d^\dagger_\rho \tilde{d}_\rho)^{(2)} \quad (2)$$

The terms $\hat{V}_{\pi\pi}$ and $\hat{V}_{\nu\nu}$, which correspond to interaction between like-boson which are given the form:

$$\hat{V}_{\rho\rho} = \frac{1}{2}\sum_{L=0,2,4} C^{(L)}_\rho([d^\dagger_\rho d^\dagger_\rho]^{(L)}.[\tilde{d}_\rho \tilde{d}_\rho]^{(L)}) \quad (3)$$

The last term $\hat{M}_{\nu\pi}$ denotes the so-called Majorana interaction, it only affects the position of mixed symmetry states and is defined as

$$\hat{M}_{\nu\pi} = \frac{1}{2}\xi_2[(d^\dagger_\nu s^\dagger_\pi - d^\dagger_\pi s^\dagger_\nu).(\tilde{d}_\nu s_\pi - \tilde{d}_\pi s_\nu)]^{(2)} + \sum_{k=1,3}\xi_k[d^\dagger_\nu d^\dagger_\pi]^{(k)}.[\tilde{d}_\nu \tilde{d}_\pi]^{(k)}. \quad (4)$$

The most general E2 transition operator can be written as [23]

$$T^{(E2)} = e_\pi Q_\pi + e_\nu Q_\nu \quad (5)$$

where e_π and e_ν are boson effective charges.

The reduced transition probability B(E2) were calculated by

$$B\left(E2{:}\, J_i \to J_f\right) = \frac{1}{2J_i+1}\, |<J_f\|T^{E2}\|J_i>|^2 \quad (6)$$

In the IBM-2, the M1 transition operator can be written as

$$T^{M1} = \sqrt{\frac{3}{4\pi}}\left(g_\pi L^{(1)}_\pi + g_\nu L^{(1)}_\nu\right) \quad (7)$$

and $L^{(1)}_\rho = \sqrt{10}\,[d^\dagger_\rho \tilde{d}_\rho]^{(1)}$

then

$$T^{M1} = 0.77[(d^\dagger d^\sim)^{(1)}_\pi - \left(d^\dagger d^\sim\right)^{(1)}_\nu] \quad (8)$$

The reduced transition probability B(M1) were calculated by

$$B(M1: J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \mid < J_f \| T^{M1} \| J_i > \mid^2 \qquad (9)$$

The mixing ratio is considered as a ratio of E2 and M1 matrix elements strength, written as [11]

$$\delta\left(\frac{E2}{M1}\right) = 0.835\, E_\gamma\, (MeV)\Delta, \qquad (10)$$

where

$$\Delta = \frac{< j_f \parallel T^{E2} \parallel j_i >}{< j_f \parallel T^{M1} \parallel j_i >} \qquad (11)$$

Table 1: Used parameters (in MeV unit), $C_\pi^0 = C_\pi^2 = C_\pi^2 = 0.02\, MeV$, and $\chi_{\nu=} 1.2$ for all isotopes.

A	N	N_ν	ε_d	$\kappa_{\pi\nu}$	χ_π	$\xi_1 = \xi_3$	ξ_2	C_ν^0	C_ν^2	C_ν^4
158	7	2	0.76	-0.09	-1.2	-0.13	0.18	-0.95	-0.95	-0.03
160	8	3	0.75	-0.07	-1.2	-0.11	0.22	-0.95	-0.78	-0.06
162	9	4	0.63	-0.09	1.2	-0.01	0.22	-0.89	-0.52	-0.12
164	10	5	0.56	-0.09	1.2	-0.08	0.04	-0.40	-0.29	0.03
166	11	6	0.46	-0.09	-1.2	-0.08	0.03	0.95	-0.42	0.50
168	12	7	0.38	-0.08	-1.2	0.15	0.06	0.95	-0.39	0.42

3- Results and discussion

We systematically show the present calculation of energy levels of the Z =72 isotopes with A = 158 to 168, and they are shown in Figs 1-6. Reproduction of the trend in the experimental data is be seen. The data are taken from Ref [24]. The parameter are adjustable to put the energy of the low-lying right, and they are given in Table 1. The energy levels have been grouped according to symmetry partition.

Examining the Fig. 1 all member of ground state in the ^{158}Hf isotope are agree well experimental ones. The model predicts the two phonon states at ~1 MeV with difference equal to 0.26 and 0.14 MeV between 0_2^+ - 2_2^+ and 2_2^+ - 4_1^+ states, respectively. The larger component of the mixed symmetry in the 2_4^+ state at 1.587 MeV has been found. as the lowest mixed symmetry state at 1.587 MeV. The 3_2^+ and 3_3^+ states at 1.909 and 2.067 MeV are coming from [N-1,1] partition.

respectively. The first scissor mode 1_1^+ and 2_{2Ms}^+ have been predicted at 1.745 and 1.813 MeV, respectively. The ^{160}Hf has $N_\pi = 5$, $N_\nu = 3$ and the energy ratio $R_{4/2} =$ E 4_1^+/ E 2_1^+equals 2.30 . The 2^+, 3^+ and 4^+ members of the gamma band are placed at 0.946, 1.416, and 1.489 MeV, and the 0^+, 2^+ and 4^+ members of the beta band are placed at 0.594, 1.181, and 1.698 MeV, respectively. From Fig. 2, we find that the lowest mixed symmetry 2_4^+ state is lower in energy than the 1_1^+ state. For ^{160}Hf the 2_5^+, 3_2^+, 3_3^+ and 1_2^+ states have $F = F_{max}$-1 configurations.

As shown in Fig. 3, the theoretical energy of the ground states bands up 10^+ state are closed to the empirical ones , for example the $E8_1^+$= 1.827 and 1.940 MeV in the IBM and EXP results. theoretical and experimental results respectively. A level 1_1^+ is determined at 1.758 MeV. The IBM-2 predicts the first 3^+ at 0.877 MeV, the second 3^+ at 1.244 MeV and the third 3^+ at 1.718 MeV, but the second and third 3^+ state are from the [N-1;1] U(6) irreducible representation. In ^{164}Hf the IBM-2 predicts the 2_1^+ at 0.216 MeV, the 2_2^+ at 0.414 MeV and 2_3^+ at 0.654 MeV are fully symmetric states. In fact, in ^{162}Hf and ^{164}Hf, the 4_1^+ state is pushed up to be relatively close to the 2_2^+ and 0_2^+states, this is a characteristic feature of the *γ-unstable* nuclei. In this calculation the excitation energy for 1_1^+= 1.302 MeV in the $^{164}Hf_{92}$ isotope is fitted with the experiment value of ^{162}Yb equal to 1.398 MeV[24].

The levels scheme of ^{166}Hf isotope has been illustrated in Fig. 5. For γ-band energy levels, the 2_γ^+ and 3_γ^+ states are predicted at 0.948 and 1.072 MeV are close to the experimental ones, at 0.810 and 1.007 MeV, respectively. The energy of 0_β^+ is equal to (1.064, 1.010) MeV while 2_β^+ (1.219,1.147) MeV and 4_β^+ (1.603,1.469) MeV in EXP and IBM results. . A mixed-symmetry character are predicted for the 2_4^+, 3_2^+, 3_3^+ and 4_4^+ states. For ^{168}Hf nucleus, it can see that the 0_β^+ and 2_β^+ states are predicted at 1.033 and 1.166 MeV close to the observed ones at 0.942 and 1.059 MeV, respectively, Though the predicted 3^+_1 levels at 1.043 MeV is very close to experimental one at 1.031MeV. According to this model such a 1+ state is member of a collective K=1 band. Up to 3 MeV ,there are only two states, 1_1^+ and 1_2^+, which are dominated by F = Fmax-1 configurations, at energies 1.931 and 2.592 MeV respectively as shown in Fig 6.

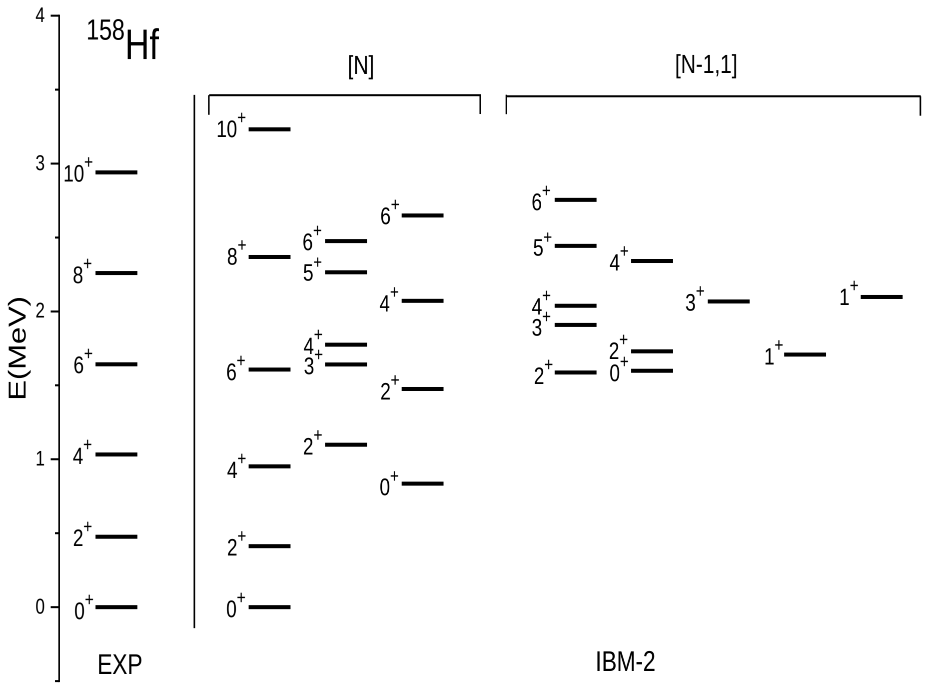

Fig. 1: Calculated and experimental energy states of the ^{158}Hf nucleus.

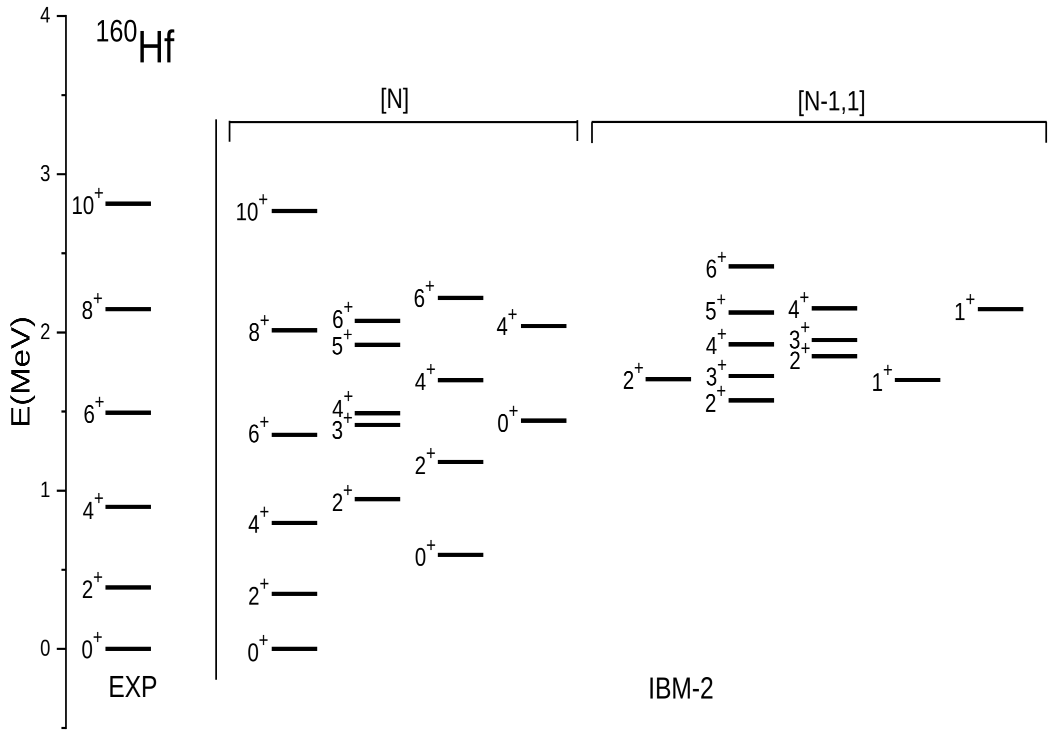

Fig. 2: Calculated and experimental energy states of the ^{160}Hf nucleus

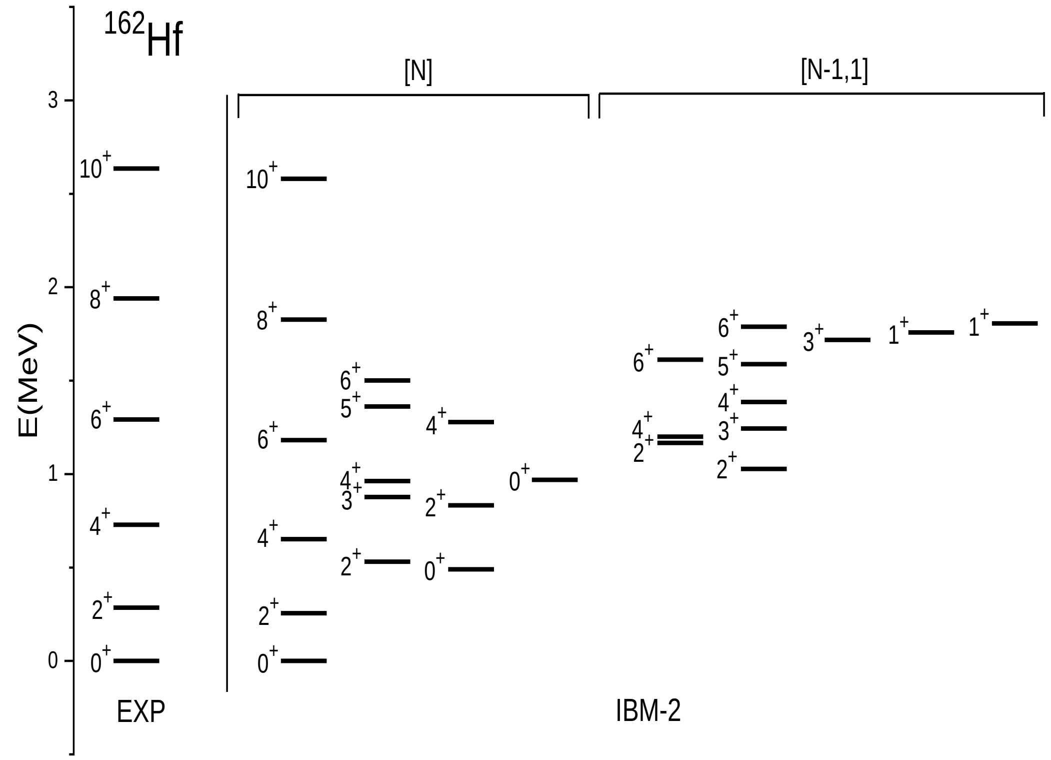

Fig. 3: Calculated and experimental energy states of the ^{162}Hf nucleus.

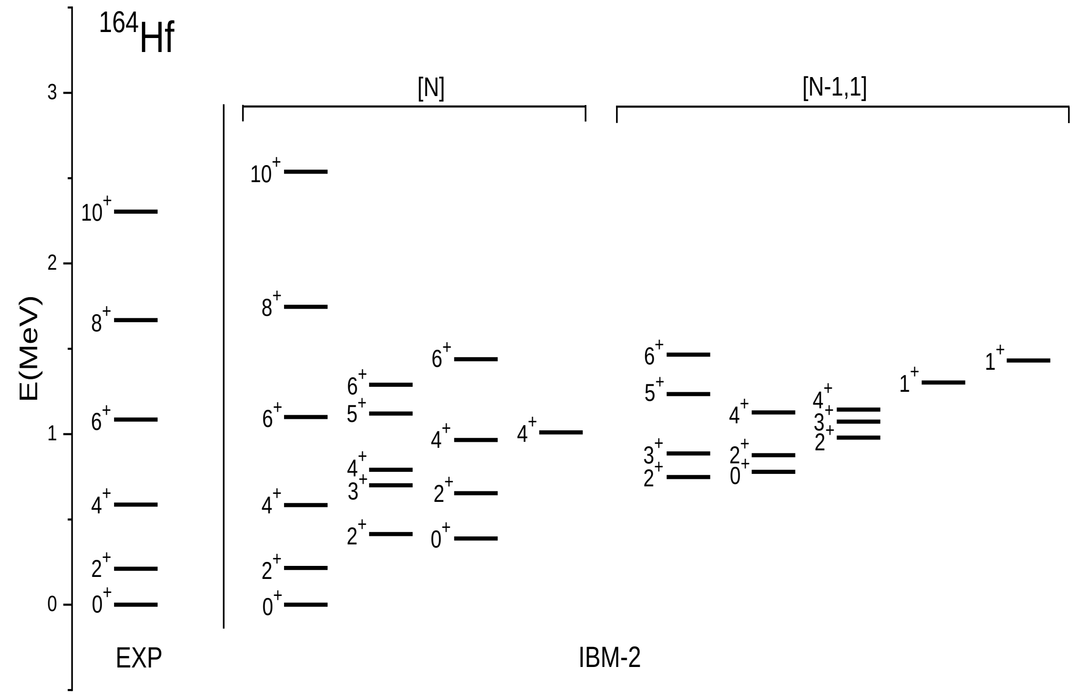

Fig. 4: Calculated and experimental energy states of the ^{164}Hf nucleus.

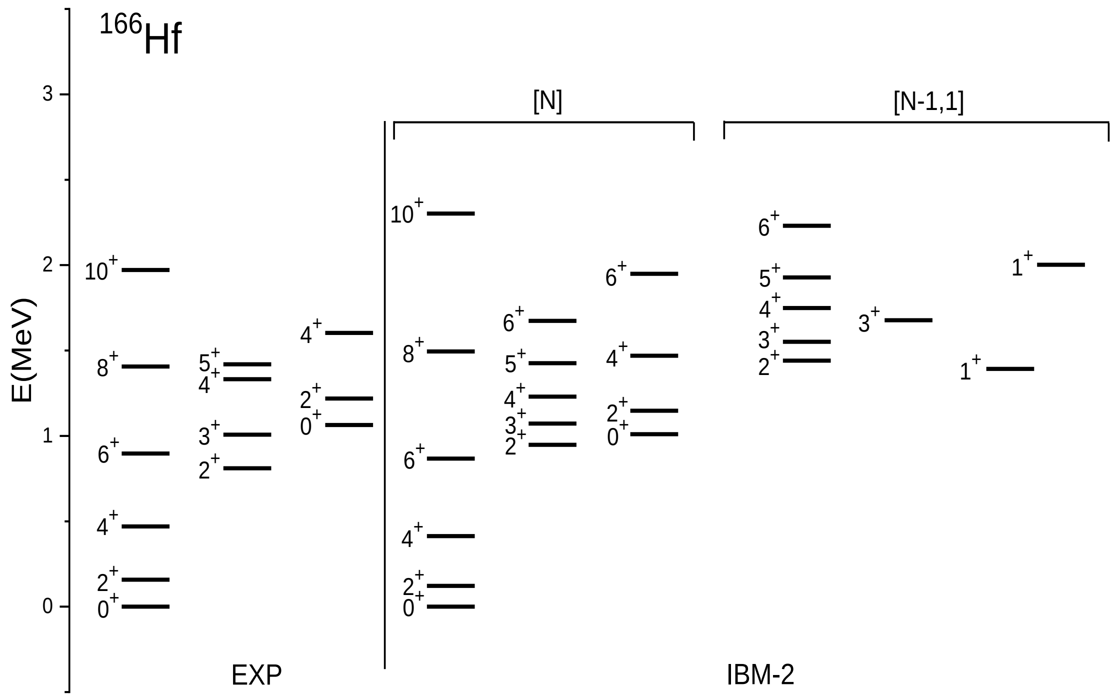

Fig. 5: Calculated and experimental energy states of the ^{166}Hf nucleus.

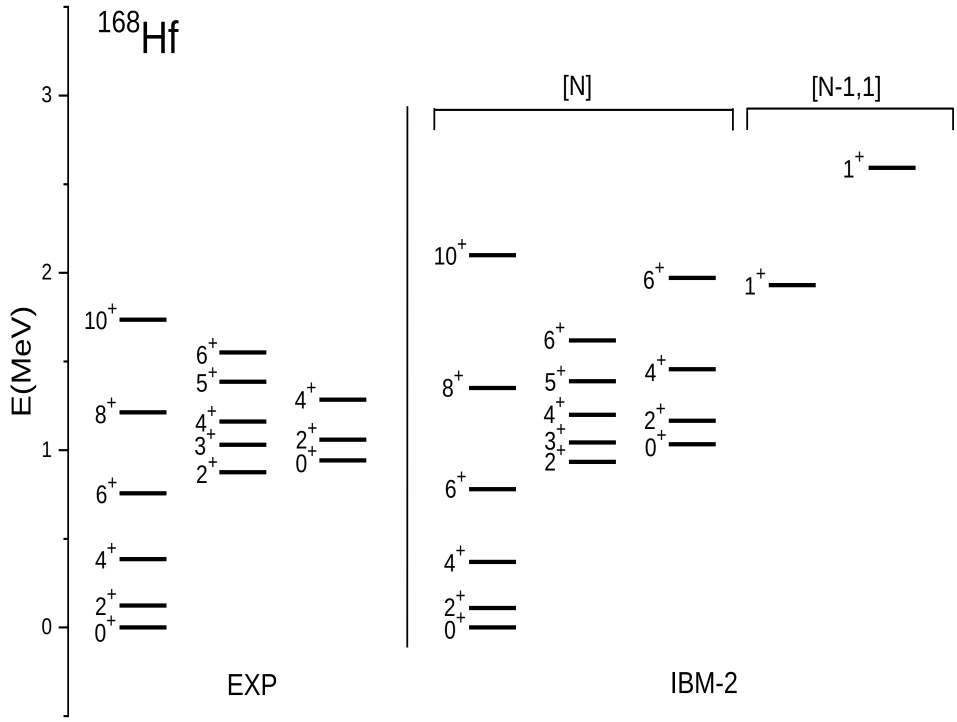

Fig. 6: Calculated and experimental energy states of the ^{168}Hf nucleus.

The change in energy of levels as a function of ξ_2 is shown in Fig. 7, with all other parameters set to their best-fit values. As can be seen in this figure, the ξ_2 has a significant impact on the energy of mixed symmetry states. We have also calculate the $R = < J \mid F^2 \mid J > / F_{max}(F_{max}+1)$ values and plotted in the Fig. 8. In order to investigate the nuclear shape, we have introduced experimental and theoretical values of energy ratios in Hf isotopes together with the values of IBM limits we have introduced experimental and theoretical values of energy ratios in Hf isotopes together with the values of IBM limits as listed in Table 2. The general features of the transition between vibrational limit U(5) in ^{158}Hf near the beginning of the closed shell and SU(3) in ^{168}Hf with moderate deformation are well reproduced by the model.

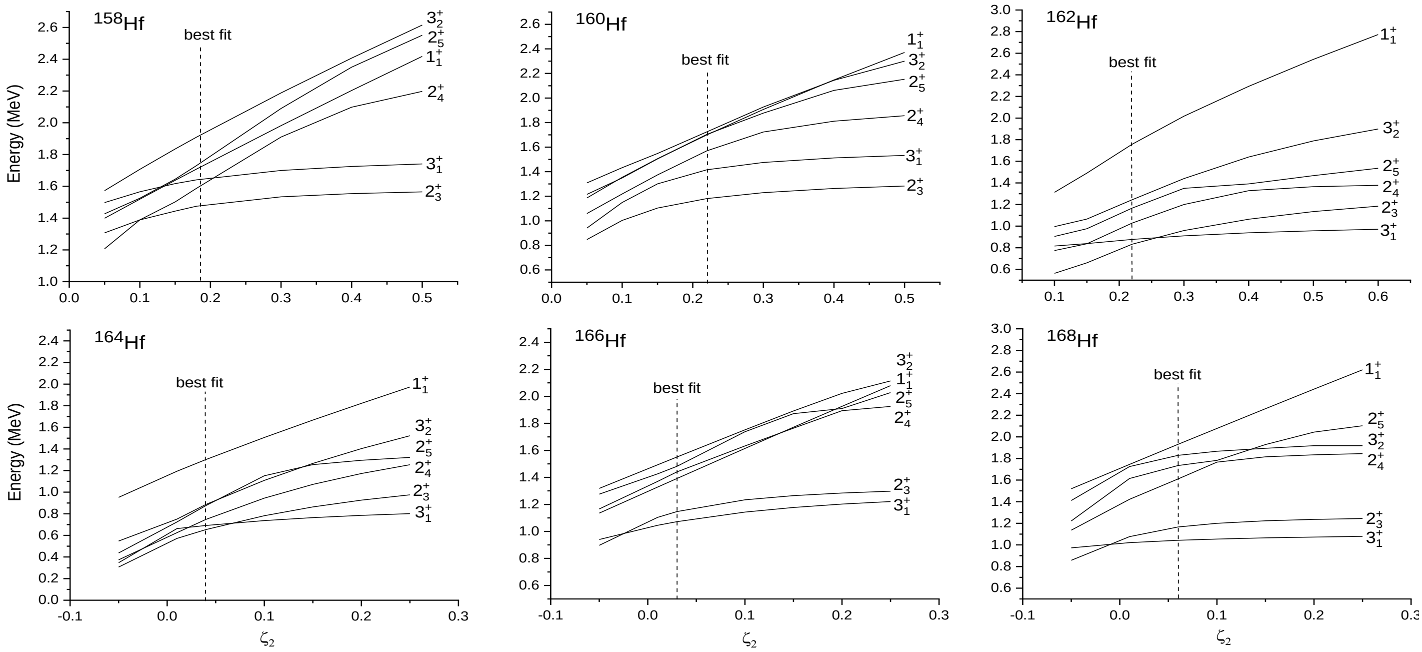

Fig. 7: Calculated energy levels as functions of Majorana parameter ξ_2.

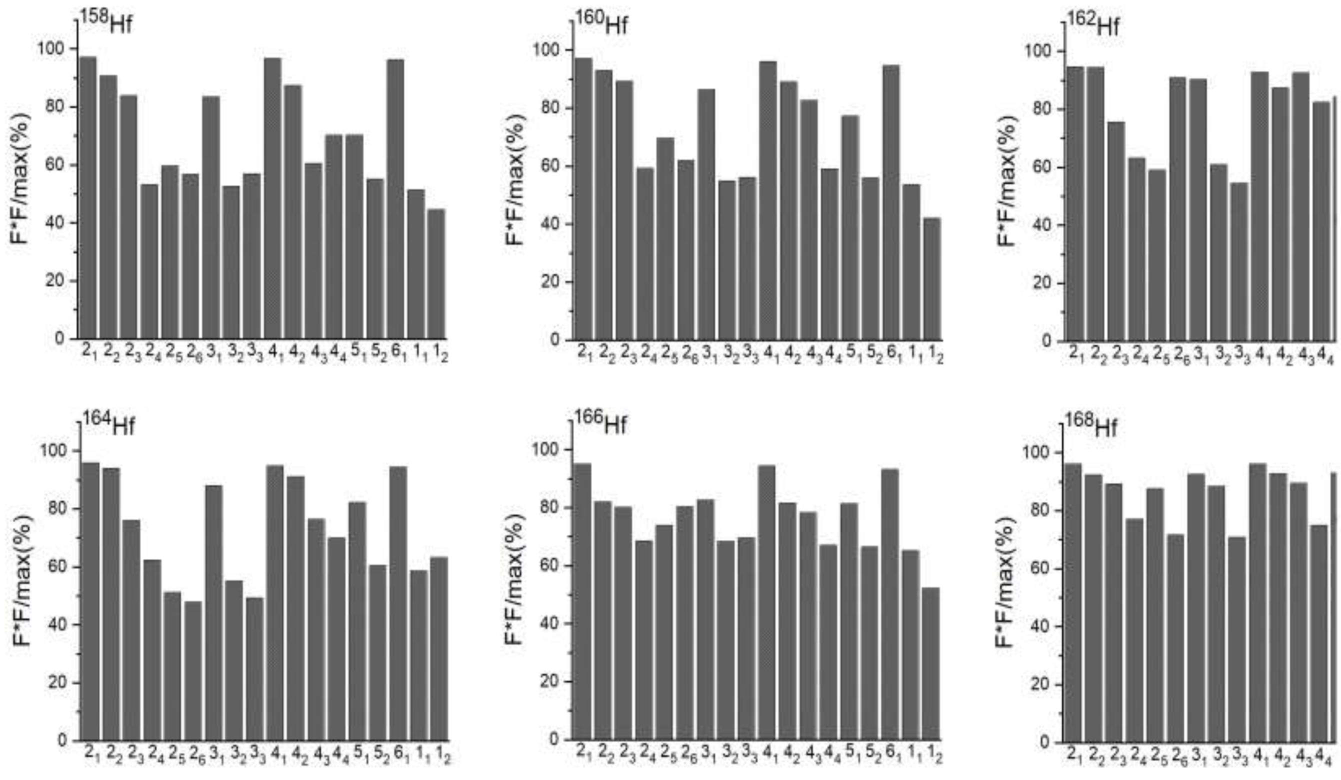

Fig. 8 : The R value of the energy states for $^{158\text{-}168}$Hf nuclei.

3- Electromagnetic Transitions

We have calculated the electromagnetic transitions E2 and M1 using equations 6 and 9. The effective boson charges e_π and e_v are estimated by normalizing the $B(E2;\ 2_1^+ \rightarrow 0_1^+)$ as listed in Table 3. For the magnetic transitions, we take g_v =0.07 μ_N , g_π = 0.15 μ_N for all isotopes. The IBM-2 predictions of the relevant transition strengths B(E2) are compared with the corresponding available experimental data as shown in Table 4. It can be seen a good agreement between the obtained B(E2) transition and the available experimental data. As shown in table 4, the calculated transition B $(E2;\ 4_1^+ \rightarrow 2_1^+)$ =0.5282, 1.0148 e^2b^2 are consistent with the experimental one 0.5493_{-906}^{+1119} , 1.0942(379) e^2b^2 for ^{164}Hf and ^{166}Hf respectively. For ^{164}Hf, the calculated value of $B(E2;\ 6_1^+ \rightarrow 4_1^+)$ is 0.6084 e^2b^2 a little lower than the experimental value of 0.9061_{-5117}^{+2238} e^2b^2, while B$(E2;\ 8_1^+ \rightarrow 6_1^+)$ is 0.6465 e^2b^2 a little higher than the experimental value of > 0.5863 e^2b^2. In the present calculation the B$(E2;\ 6_1^+ \rightarrow 4_1^+)$ equal to (1.0674, 1.1974(704)) for the ^{166}Hf isotope in the (IBM, EXP) results, and B$(E2;\ 4_1^+ \rightarrow 2_1^+)$ and B$(E2;\ 6_1^+ \rightarrow 4_1^+)$ are equal to (1.1874, 1.3554(660)) and (1.2585, 1.5681(990)) for the ^{168}Hf isotope, respectively. The calculated results of the B(M1) values are shown in Tables 5. We observed a large M1 strength for $1_1^+ \rightarrow 0_1^+$ transition, *B(M1)* = 0.004016μ_N^2 in comparison with B(M1; $1_1^+ \rightarrow 2_1^+$) = 0.002374 μ_N^2 in ^{168}Hf isotope.. The *M1* decay of 3_2^+ to 2_1^+ and 2_2^+ states are dominant for all isotopes.

Table 2: Experimental and theoretical values of energy ratios in Hf isotopes.

A	$E4_1^+/E2_1^+$		$E2_2^+/E2_1^+$		$E0_2^+/E4_1^+$		$E0_2^+/E2_1^+$	
	EXP	IBM-2	EXP	IBM-2	EXP	IBM-2	EXP	IBM-2
158	2.17	2.30		2.66		0.63		1.46
160	2.30	2.28		2.71		0.74		1.70
162	2.55	2.55		2.08		0.75		1.92
164	2.78	2.70		1.91		0.66		1.79
166	2.97	3.38	5.12	7.77	2.26	2.44	6.73	8.27
168	3.10	3.36	7.05	8.49	2.44	2.79	7.59	9.39
U(5)		2.00		2.00		1.00		2.00
O(6)		2.50		>2.00		~1.00		4.50
SU(3)		3.30		3.00		≫1.00		≫2.00

Table 3: The effective boson charges e_v and e_π used in IBM-2 for the calculation of B(E2) transition probabilities in e^2b^2 unit.

A	158	160	162	164	166	168
e_v	0.08	0.08	0.08	0.08	0.08	0.08
e_π	0.11	0.13	0.15	0.18	0.19	0.20

Table 4: The absolute B(E2) values calculated in e^2b^2, compared with the available experimental data.

$J_i^+ \rightarrow J_f^+$	^{158}Hf		^{160}Hf		^{162}Hf	
	EXP	IBM	EXP	IBM	EXP	IBM
$2_1 \rightarrow 0_1$		0.1146		0.1773	0.2671(209)	0.2578
$4_1 \rightarrow 2_1$		0.2010		0.3280		0.3744
$6_1 \rightarrow 4_1$		0.2356		0.3879		0.4223
$8_1 \rightarrow 6_1$		0.2350		0.3920		0.4298
$2_2 \rightarrow 0_1$		0.0037		0.0070		0.0189
$2_3 \rightarrow 0_1$		0.0010		0.0008		0.0006
$2_2 \rightarrow 2_1$		0.0470		0.0512		0.3049
$2_3 \rightarrow 2_1$		0.0001		0.0071		0.0187
$3_1 \rightarrow 2_1$		0.0055		0.0050		0.0277
$3_1 \rightarrow 4_1$		0.0123		0.0090		0.1301
$3_2 \rightarrow 2_3$		0.0018		0.0001		0.0015
$1_1 \rightarrow 2_1$		0.0012		0.0026		0.0002
$1_1 \rightarrow 2_2$		0.0014		0.0001		0.0068
$1_1 \rightarrow 2_3$		0.0459		0.0104		0.0021
	^{164}Hf		^{166}Hf		^{168}Hf	
$J_i^+ \rightarrow J_f^+$	EXP	IBM	EXP	IBM	EXP	IBM
$2_1 \rightarrow 0_1$	0.3624(373)	0.3607	0.6931(379)	0.7266	0.8482(385)	0.8478
$4_1 \rightarrow 2_1$	0.5493^{+1119}_{-906}	0.5282	1.0942(379)	1.0148	1.3445(660)	1.1874
$6_1 \rightarrow 4_1$	0.9061^{+2238}_{-5117}	0.6084	1.1974(704)	1.0674	1.5681(990)	1.2585
$8_1 \rightarrow 6_1$	>0.5863	0.6465	1.5168(1625)	1.0266	1.9265(2752)	1.2328
$2_2 \rightarrow 0_1$		0.0366		0.0029		0.0055
$2_3 \rightarrow 0_1$		0.0049		0.0005		0.0003
$2_2 \rightarrow 2_1$		0.4502		0.0016		0.0044
$2_3 \rightarrow 2_1$		0.0222		0.0002		0.0004
$3_1 \rightarrow 2_1$		0.0667		0.0053		0.0101
$3_1 \rightarrow 4_1$		0.1773		0.0001		0.0002
$3_2 \rightarrow 2_3$		0.0019		0.0183		0.0606
$1_1 \rightarrow 2_1$		0.0016		0.0052		0.0162
$1_1 \rightarrow 2_2$		0.0062		0.0058		0.0077
$1_1 \rightarrow 2_3$		0.0005		0.0317		0.0036

Table 5: The absolute B(M1) values calculated in μ_N^2 unite, for 158-168Hf isotopes.

$J_i^+ \rightarrow J_f^+$	^{158}Hf	^{160}Hf	^{162}Hf	^{164}Hf	^{166}Hf	^{168}Hf
$2_2 \rightarrow 2_1$	0.000021	0.000001	0.000102	0.000307	0.000025	0.000022
$2_3 \rightarrow 2_1$	0.000796	0.000105	0.000056	0.000042	0.000157	0.000002
$2_4 \rightarrow 2_1$	0.000036	0.000181	0.000025	0.000051	0.000016	0.000548
$2_5 \rightarrow 2_1$	0.001279	0.001144	0.000009	0.000003	0.000000	0.001492
$2_5 \rightarrow 2_3$	0.000290	0.000247	0.000022	0.000002	0.000000	0.000006
$3_1 \rightarrow 2_1$	0.000165	0.000038	0.000036	0.000065	0.000024	0.000021
$3_2 \rightarrow 2_1$	0.000088	0.000181	0.000008	0.000005	0.001811	0.000023
$3_2 \rightarrow 2_2$	0.000471	0.000328	0.000094	0.000066	0.000001	0.000006
$3_2 \rightarrow 2_3$	0.000031	0.000055	0.000002	0.000001	0.000005	0.000019
$3_1 \rightarrow 4_1$	0.000443	0.000075	0.000187	0.000370	0.000035	0.000034
$1_1 \rightarrow 0_1$	0.000568	0.000652	0.000159	0.000372	0.003907	0.004016
$1_1 \rightarrow 2_1$	0.000746	0.000922	0.000449	0.000136	0.002302	0.002374
$1_1 \rightarrow 2_2$	0.000758	0.000884	0.000059	0.000275	0.000379	0.000020
$1_1 \rightarrow 2_3$	0.000008	0.000001	0.000009	0.000254	0.000030	0.000013

The decays of the $2_2^+, 2_3^+, 2_4^+$ and 2_5^+ levels to the 2_1^+ states are obtained to be of mixed E2/M1 multipolarity. The mixing ratio δ(E2/M1) for the selected transitions for Hf isotopes are presented in Table 6. In the present calculation, the 1_1^+ state has dominant *M1* decays to 2_3^+ with a mixing ratio δ(E2/M1) = +17.996, and the 1_2^+ state has dominant *M1* decays to 2_3^+ with a mixing ratio δ(E2/M1) = -0.953, in ^{158}Hf isotope. For ^{168}Hf isotope, we observe the δ($2_3^+ \rightarrow 2_1^+$) is equal to -8^{+4}_{-10} and -11.190 and the δ($3_1^+ \rightarrow 2_1^+$) is equal to $+11^{+13}_{-4}$ and +16.769 in the experimental and IBM-2 result respectively. Accordingly, δ values for ($3_1^+ \rightarrow 4_1^+$) transitions are positive in all Hf isotopes investigated in our works.

Table 6: Mixing ratios δ(E2/M1) in eb/μ_N for 158-168Hf, compared with the available experimental data.

A	158	160	162	164	166	168	
$J_i^+ \rightarrow J_f^+$	IBM	IBM	IBM	IBM	IBM	EXP	IBM
$2_2 \rightarrow 2_1$	+27.642	-126.33	+12.926	+6.358	+5.768	-10^{+3}_{-9}	+9.280
$2_3 \rightarrow 2_1$	-0.179	+5.912	+9.230	+8.699	-1.046	-8^{+4}_{-10}	-11.190
$2_4 \rightarrow 2_1$	+15.307	+6.088	+6.395	+4.358	+28.910		-3.832
$2_5 \rightarrow 2_1$	-2.115	-1.494	+9.645	+4.822	+25.645		-0.678
$3_1 \rightarrow 2_1$	+5.911	+10.422	+15.234	+13.144	+12.692	$+11^{+13}_{-4}$	+16.769
$3_2 \rightarrow 2_1$	+1.665	+5.683	+21.471	+24.988	+2.258		-5.023
$3_1 \rightarrow 4_1$	+3.064	+5.882	+4.978	+2.156	+0.880	>10	+1.419
$1_1 \rightarrow 2_1$	-1.334	-1.910	-0.906	+3.173	-1.597		-3.986
$1_1 \rightarrow 2_2$	-0.703	+5.450	-11.269	+3.601	+1.482		+16.444
$1_1 \rightarrow 2_3$	+17.996	+58.670	-12.203	+0.768	-7.001		-10.995
$1_2 \rightarrow 2_3$	-0.953	+8.418	-0.718	-1.338	-1.376		-5.749

5- CONCLUSION

In summary, we have investigated the levels scheme and the symmetry properties of the even-even $^{158\text{-}168}$Hf nuclei. The characteristics of the mixed-symmetry states 2^+_{ms} and 1^+_{ms} for even-even $^{158\text{-}168}$Hf have been identified. The calculation indicates that 3^+_2 and 3^+_3 states belong to the mixed-symmetry states for all isotopes except ^{168}Hf, and only a few MS states have been identified in non-rotational nuclei. The mixing ratios of some gamma to ground state band transitions were determined. The evolution of the nuclear shape from U(5) to SU(3) limit of Hafnium isotopes were found in the model results.

REFERENCES

[1] E.A McCutchan, N.V Zamfir, R.F. Casten, Phys. Rev. C **71**, 034309 (2005)

[2] F.H. Al-Khudair, G.L. Long, Y Sun, Phys. Rev. C **77**, 034303 (2008).

[3] K. Heyde, J.L. Wood, Rev. Mod. Phys. **83**, 1467 (2011).

[4] K. Nomura, R. Rodriguez-Guzman, L. M. Robledo, Phys. Rev. C **101**, 014306 (2020).

[5] H.N. Qasim, F.H. Al-Khudair, Nucl. Phys. A **1002**,121962 (2020).

[6] K. Nomura, R Rodriguez-Guzman, L.M. Robledo, J.E. Garcia-Ramos, Phys. Rev. C **103**, 044311 (2021).

[7] F.H. Al-Khudair, Phys. Rev. C **80**, 014306 (2020).

[8] D. Zhang, C Mu, Sci. China Phys. Mech. Astron. **60**, 042011 (2017).

[9] T. Thomas et al. Nucl. Phys. A **947**, 303 (2016).

[10] U. Milkau, D. Bohle, A.Richter, Nucl. Phys. A **499**, 517 (1989).

[11] B. Decroix, C. De Coster, K. Heyde, A. M. Oros, J. De Beule, Phys. Rev. C **58**, 232(1998)

[12] P. von Brentano, N. Pietralla, C. Fransen, A. Gade, A. Gelberg, U. Kneissl, T. Otsuka, H. H. Pitz, V.Werner, News **99**, 255 (2002).

[13] R.F. Casten, N.V. Zamfir, Phys. Rev. Lett. **87**, 052503 (2001).

[14] J.E. Garcia-Ramos, J.M. Arias, J. Barea, A. Frank, Phys. Rev. C **68**, 024307 (2003).

[15] K.S. Krane, Phys. Rev. C **8** 1494 (1973).

[16] A.E. Stuchbery, Nucl. Phys. A **589**, 222 (1995).

[17] D.L. Zhang, B.G. Ding, Commun. Theor. Phys. **51**, 126 (2009).

[18] B. Bochev, S. Iliev, R. Kalpakchieva, S.A. Karamian, T. Kutsarova, E. Nadjakov, T. Venkova, Nucl. Phys. A **282**, 159 (1977).

[19] M. Rudigier, K. Nomura, M. Dannhoff, R-B. Gerst, J. Jolie, N. Saed-Samii, S. Stegemann, J-M. Regis, L. M. Robledo, R. Rodriguez-Guzman, A. Blazhev, Ch. Fransen, N. Warr, K. O.Zell, Phys. Rev. C **91** 044301 (2015).

[20] E.A. McCutchan, N.V. Zamfir, R.F. Casten, H. Ai, H. Amro, M.Babilon, D. S. Brenner, G. Gurdal, A. Heinz, R.O. Hughes, D.A. Meyer, C. Plettner, J. Qian, J.J. Ressler, N.J. Thomas, V. Werner, E. Williams, R. Winkler, Phys. Rev. C **73**, 034303 (2006).

[21] X.Q. Yang, L.J. Wang, J. Xiang, X.Y. Wu, Z.P. Li, Phys. Rev. C **103**, 054321 (2021).

[22] Y. Beom Choi, Chang-Hwan Lee, Myeong-Hwan Mun, Youngman Kim, Phys. Rev. C **105**, 024306 (2022).

[23] F. Iachello, A. Arima, The Interacting Boson Model, Cambridge University Press, Cambridge, England, (1987).

[24] ENSDF, http:// www.nndc.bnl.gov/ensdf, Nuclear data Sheet (2022).

Journal of Basrah Researches (Sciences) **48**(2), 44 (2022)
DOI: https://doi.org/10.56714/bjrs.48.2.5

تركيب المستويات المنخفضة لإنويه الهافنيوم $^{158-168}Hf$

سلطان فالح حسين الخضير[*] ، حسين ناصر قاسم

قسم الفيزياء، كلية التربية للعلوم الصرفة، جامعة البصرة، البصرة، العراق.

معلومات البحث

الاستلام	12 حزيران 2022
القبول	04 ايلول 2022
النشر	30 كانون الاول 2022

الكلمات المفتاحية

المستويات ذات المتماثلة المختلط ، انموذج البوزونات المتفاعلة، نسب الخلط، أنوية الهافنيوم.

الملخص

تم دراسة خصائص المستويات ذات المتماثلة المختلط المنخفضة لانوية الهافنيوم $^{158-168}Hf$ في اطار انموذج البوزونات المتفاعلة. قورنت نتائج مستويات الطاقة واحتمالية الانتقالات الكهرومغناطيسية ونسب الخلط مع القيم العملية المتوفرة . تم اختبار تأثير معاملات ماجرونا عل مستويات الطاقة. وجد ان المستويات 2_{ms}^{+} و 1^{+}هي او أوطئ المستويات المتماثلة المختلط في أنوية $^{158-164}Hf$ الاهتزازية و $^{166,168}Hf$ الدورانية.

Citation: F.H. Al-Khudair, H.N. Qasim, J. Basrah Res. (Sci.) **48**(2), 44 (2022). DOI:https://doi.org/10.56714/bjrs.48.2.5

*Corresponding author email : falih9@gmail.com

ISSN: 1817-2695 (Print); 2411-524X (Online)
Online at: https://bjrs.uobasrah.edu.iq