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Chebyshev-Homotopy Perturbation Method for Studying the Flow and Heat Transfer of a Non-Newtonian Fluid Flow on the Turbine Disk

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ABSTRACT

In this investigation, a new method for studying the effect of non-Newtonian fluid on the flow and temperature distribution when cooling the turbine disk is presented. The new method is based on the homotopy perturbation method developed with the Chebyshev series. The results of the proposed method were compared with the results obtained using numerical methods in previous literature to ensure the validity of the method, as it showed good agreement. The effect of several physical parameters on flow velocity and temperature diffusion, such as the Reynolds number, cross viscosity parameter, Prandtl number, and power law, was explored. The results obtained using the proposed method were more accurate than other methods used to solve the current problem. Moreover, figures and error tables show the new method's efficacy and efficiency.

1. Introduction

The application of non-Newtonian fluids is an important and widespread issue. One of the most important of these applications is the flow of non-Newtonian viscous elastic fluid in an isoaxial channel with a porous wall. Because the goal of the application is to reduce the heat generated on the turbine disk, researchers paid great attention to it. As an example, Dogonchi and Ganji [1] used a novel technique depending on the Duan-Rach methodology in order to solve the problem of turbine cooling application. Sepasgozar et al. [2] studied non-Newtonian fluid flow in a porous channel using the differential transformation method (DTM). Their findings supported the efficacy of their methodology, with comparisons to numerical approaches demonstrating high agreement. Mirgolbabaee and colleagues [3] presented Akbari-Ganji's approach for determining the approximate solutions of the nonlinear equations that describe the flow of a non-Newtonian fluid to the problem of turbine cooling. A comparative evaluation with the fourth-order Runge-Kutta technique revealed a strong agreement in their results. Singh and Yadav [4] employed the perturbation method to findthe approximate solution to the heat transfer and momentum equations of the non-Newtonian fluid flow.

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This study focused on the effect of some parameters, as it showed that the variation between the Reynolds number and the velocity is direct, and similar between the Prandtl number and the temperature. Sheikhzadeh et al.[5] applied the least square and Galerkin methods to solve governing equations of a non-Newtonian laminar fluid flow in a porous bounding wall. The results of the comparison with the Runge–Kutta method of fourth-order showed a good congruence in addition to the distinction of the Galerkin method over the least square method in terms of simplicity of steps and fewer calculations. Akinshilo et al.[6] used the variation iteration and homotopy perturbation methods to find an approximate solution for a non-Newtonian viscoelastic fluid flow in an axisymmetric channel. This study discussed the effect of heat generated on the turbine disks during flow as well as verified the validity of the results of the analytical solutions by comparing them with the results of numerical methods. Al-Griffi and Al-Saif[7] introduced a new method based on the homotopy perturbation method and the Yang transform to solve a non-Newtonian viscoelastic fluid flow in an axisymmetric channel. This study showed the effect of some important parameters on the governing equations, and it showed a good agreement with the results of numerical methods.

Semi-analytical methods have appeared as the preferred approach for finding analytical approximations to complex problems containing nonlinear terms [7-14]. This preference arises from the challenges associated with obtaining exact solutions using conventional analytical techniques, as well as the accuracy and convergence issues encountered with numerical methods. Consequently, a significant number of researchers and engineers (as mentioned earlier) have turned to semi-analytical methods to investigate such problems and gain deeper insights into their intricacies. Among these methods, the homotopy perturbation method appeared particularly noteworthy, drawing the interest of numerous scientists and finding application in solving a wide array of complex problems, including the one under consideration [6]. In 2010, Aminikhah and Hemmatnezhad [15] introduced a refined version of this approach, termed the new homotopy perturbation method, which was applied to derive approximate solutions for the quadratic Riccati differential equation. This novel method is discrete by its simplicity and ease of mathematical computation, making it a popular choice in various studies [16–20]. The two basic ideas that underpin the method's progression are as follows: first, the first approximation should be defined as a power series; second, all iterations should be set to zero except for the initial one. Therefore, it is evident that the power series assumes a pivotal position in the advancement of the novel homotopy perturbation technique, facilitating the acquisition of approximate solutions for nonlinear equations in the format of the Taylor expansion.

In the present work, we seek to improve accuracy and convergence in analytic approximation solutions by using a series that is more effective than the Taylor expansion to create a novel homotopy perturbation technique. Of all the power series that are accessible, the Chebyshev series stands out as the most important one since it has several benefits. The orthogonal Chebyshev polynomials on which this series is built are the building blocks for approximating known functions. Due to the faster rate of convergence, the Chebyshev series outperforms the Taylor series in terms of efficiency and accuracy since when the Taylor error moves away from the convergence point, it increases rapidly [21]. Owing to these special properties, the Chebyshev series has been widely applied in a wide range of academic studies, acting as a primary or auxiliary tool for addressing nonlinear equations and yielding very precise approximations. For instance, Hamada[22] introduced a novel approach to approximate solutions for complex linear/nonlinear systems of point kinematic equations, employing the first type of shifted Chebyshev series. Arushanyan and Zaletkin [23] elucidated a technique for resolving canonical second-order ordinary differential equations, grounded on an approximation of the solution to the Cauchy problem and its derivatives, utilizing the partial sum of the shifted Chebyshev series. Ali et al.[24] utilized Chebyshev's series to devise a numerical method for tackling nonlinear integral-differential equations of fractional order. Wang et al.[25] proposed a new collocation method for solving two-dimensional partial differential equations, leveraging the localized Chebyshev collocation method to achieve spectral accuracy by computing unknown functions at each node. Zaletkin[26] relied on the shifted Chebyshev series and a Markov quadrature formula to devise an approximate technique for solving second-order ordinary differential equations. Izadi et al. [27] introduced a common approximation method, utilizing the Chebyshev collocation technique to approximate the space variables of the Berger equation. Zaletkin [28] employed a technique that involved the utilisation of shifted Chebyshev series in conjunction with a Markov quadrature formula in order to solve systems of ordinary differential equations. Duan and Jing [29] based on shifted Chebyshev polynomials for addressing the initial and boundary value problems associated with the fractional diffusion equation.

The impetus behind our introduction to this study stems from the pivotal role of the Chebyshev series, as highlighted in the historical context. Furthermore, current research is centered on advancing semi-analytical methods to surmount the challenges inherent in these approaches, as well as addressing the time-consuming nature of numerical methods. These factors collectively motivated us to devise a novel method aimed at mitigating these challenges and reducing the iteration count typically encountered when solving nonlinear equations.

So, this study's main accomplishment is the creation of a novel, accurate, and efficient analytical approximation approach that is used for the first time to calculate an approximate solution to the current problem. This new method termed the Chebyshev-homotopy perturbation method (CHPM), is devised by integrating the efficient Chebyshev series with the new homotopy perturbation approach. The study delves into the effects of Reynolds and Prandtl numbers, along with the power-law index, on velocity and heat transfer. A comparison with existing methods reported in the literature was conducted to confirm the efficiency of the novel method, and a perfect agreement was obtained. The key results obtained through the new method underscore its efficiency, success, and high accuracy compared to existing methods used to deal with the present problem.

2. Fundamental concepts of the CHPM

The novel method is mainly based on the use of the Chebyshev series in the new homotopy perturbation technique. In this section, we will mention some basic concepts of the Chebyshev series, the new homotopy perturbation algorithm, and then discuss our new algorithm for finding approximate solutions to nonlinear differential equations.

2.1. Chebyshev series

Assume that $f(x)$ is a continuous function in the interval $[a,b] \in \mathbb{R}$. Then the the first kind Chebyshev series of $f(x)$ is given by[30-32]:

$$f(x) = {\sum_{i=0}^{\infty}}' A_i\, C_i\left(\frac{2x-b-a}{b-a}\right), \tag{1}$$

where $'$ sign indicates that the coefficient of $C_0(x)$ must be reduced by half, $C_i(x) = \cos(i\,t)$ with $t = \cos^{-1}(x)$ and $A_i = \frac{2}{\pi}\int_{-1}^{1} \frac{u\left(\frac{b+a}{2}+\frac{b-a}{2}x\right) C_i(x)}{\sqrt{1-x^2}}\,dx$.

Chebyshev polynomials of the first kind, defined within the interval $[-1, 1]$, can be obtained through the following recurrence relation:

$$C_i(x) = 2x\, C_{i-1}(x) - C_{i-2}(x) \;\; , i = 2,3,..., \tag{2}$$

where $C_0(x) = 1$ and $C_1(x) = x$.

Moreover, the finite summation of the powers of x can be employed to obtain it as follows:

$$C_i(x) = \sum_{j=0}^{\left\lfloor \frac{i}{2} \right\rfloor} d_j^{(i)} x^{i-2j}, \tag{3}$$

where $d_j^{(i)} = (-1)^j\, 2^{i-2j-1} \frac{i}{i-j} \binom{i-j}{j}$.

On the contrary, the power of x can be expressed as a finite summation of the first kind Chebyshev polynomials using the following formula:

$$x^i = 2^{1-i} {\sum_{j=0}^{\left\lfloor \frac{i}{2} \right\rfloor}}' \binom{i}{j} C_{i-2j}(x). \tag{4}$$

In the present study, the range of x is $[0, 1]$. Consequently, the recurrence relation was derived to determine the shifted Chebyshev polynomials $T_i^*(x)$ of the first kind, utilizing the aforementioned recurrence relation in Eq. (2), as follows:

$$T_i^*(x) = 2(2x-1)\, T_{i-1}^*(x) - T_{i-2}^*(x), i = 2,3, \ldots , \tag{5}$$

where $T_0^*(x) = 1$ and $T_1^*(x) = 2x - 1$.

We can also express the powers of x as the term of the first kind shifted Chebyshev polynomials as the following:

$$x^i = {\sum_{j=0}^{i}}' 2^{1-2i} \binom{2i}{i-j} T^*{}_j(x) \ . \quad (6)$$

The Chebyshev polynomials possess significant mathematical relationships.

Let $z = \frac{2x-b-a}{b-a}$, $[a,b] \in \mathbb{R}$ and $i, j \in \mathbb{N} \cup \{0\}$ then the following relations are true:

1. The multiplication relation is

$$C_i(z)\, C_j(z) = \frac{1}{2}\left(C_{i+j}(z) + C_{|i-j|}(z)\right). \quad (7)$$

2. The derivation relation is

$$\frac{d}{dx}\left(C_i(z)\right) = \frac{4i}{b-a} {\sum_{j=0}^{\left\lfloor \frac{i-1}{2} \right\rfloor}}' C_{i-1-2j}(z). \quad (8)$$

3. The integration relation is

$$\int C_i(z)\, dx = \begin{cases} \frac{b-a}{4}\left(\frac{C_{i+1}(z)}{i+1} - \frac{C_{i-1}(z)}{i-1}\right), & i \geq 2 \\ \frac{b-a}{8} C_2(z), & i = 1 \\ \frac{b-a}{2} C_1(z), & i = 0 \end{cases} . \quad (9)$$

Table 1. The powers of x and the shifted Chebyshev polynomials express one in terms of the other.

$T_0^*(x) = 1$	$1 = T_0^*(x)$
$T_1^*(x) = 2x - 1$	$x = \frac{1}{2}\left(T_0^*(x) + T_1^*(x)\right)$
$T_2^*(x) = 8x^2 - 8x + 1$	$x^2 = \frac{1}{8}\left(3T_0^*(x) + 4T_1^*(x) + T_2^*(x)\right)$
$T_3^*(x) = 32x^3 - 48x^2 + 18x - 1$	$x^3 = \frac{1}{32}\left(10T_0^*(x) + 15T_1^*(x) + 6T_2^*(x) + T_3^*(x)\right)$
$T_4^*(x) = 128x^4 - 256x^3 + 160x^2 - 32^x + 1$	$x^4 = \frac{1}{128}\left(35T_0^*(x) + 56T_1^*(x) + 28T_2^*(x) + 8T_3^*(x) + T_4^*(x)\right)$
$T_5^*(x) = 512x^5 - 1280x^4 + 1120x^3 - 400x^2 + 50x - 1$	$x^5 = \frac{1}{512}\left(252T_0^*(x) + 210T_1^*(x) + 120T_2^*(x) + 45T_3^*(x) + 10T_4^*(x) + T_5^*(x)\right)$

2.2. The new homotopy perturbation method

To elucidate the procedure of this approach, let's suppose that the nonlinear equation in the formula of operators, and as follows[15-20]:

$$L\big(f(x)\big) + R\big(f(x)\big) + N\big(f(x)\big) + g(x) = 0\,, \qquad x \in [a,b], \quad (10)$$

where N is the nonlinear operator, L and R are the linear operators, and $g(x)$ is a known function.

By employing the principle of homotopy, we obtain:

$$H(f,p) = (1-p)\Big(L\big(f(x)\big) - f^*(x)\Big) + p\Big(L\big(f(x)\big) + R\big(f(x)\big) + N\big(f(x)\big) + g(x)\Big) = 0 \quad , \quad (11)$$

or

$$H(f,p) = L\big(f(x)\big) - f^*(x) + pf^*(x) + p\Big(R\big(f(x)\big) + N\big(f(x)\big) + g(x)\Big) = 0, \quad (12)$$

where $p \in [0,1]$ is an embedding parameter and $f^*(x)$ is an initial approximation of the solution of Eq.(10). From Eq.(12)

$$L\big(f(x)\big) = f^*(x) - pf^*(x) - p\Big(R\big(f(x)\big) + N\big(f(x)\big) + g(x)\Big), \quad (13)$$

$$f(x) = L^{-1}\big(f^*(x)\big) - pL^{-1}\big(f^*(x)\big) - pL^{-1}\Big(R\big(f(x)\big) + N\big(f(x)\big) + g(x)\Big), \quad (14)$$

Suppose that $f(x) = \sum_{i=0}^{\infty} p^i u_i(x)$ and $f^*(x) = \sum_{i=0}^{\infty} c_i v_i(x)$ then, we obtain

$$\sum_{i=0}^{\infty} p^i u_i(x) = L^{-1}(\sum_{i=0}^{\infty} c_i v_i(x)) - pL^{-1}(\sum_{i=0}^{\infty} c_i v_i(x)) - pL^{-1}\left(R(\sum_{i=0}^{\infty} p^i u_i(x)) + N(\sum_{i=0}^{\infty} p^i u_i(x)) + g(x)\right), \quad (15)$$

Equating the coefficients of the powers of p, we get

$$\left.\begin{array}{ll} p^0: & u_0 = L^{-1}(\sum_{i=0}^{\infty} c_i v_i(x)) \\ p^1: & u_1 = -L^{-1}(\sum_{i=0}^{\infty} c_i v_i(x)) - L^{-1}\big(R(u_0) + N(u_0) + g(x)\big) \\ p^2: & u_2 = -L^{-1}\big(R(u_0, u_1) + N(u_0, u_1)\big) \\ p^i: & u_i = -L^{-1}\big(R(u_0, u_1, \dots, u_{i-1}) + N(u_0, u_1, \dots, u_{i-1})\big), \quad i = 3,4, \dots \end{array}\right\}, \quad (16)$$

Now, we assume that $u_1 = 0$, and find the values of the unknown coefficient c_i. Thus, the exact solution becomes as follows**:**

$$f(x) = u_0 = L^{-1}(\sum_{i=0}^{\infty} c_i v_i(x)). \quad (17)$$

2.3. The CHPM algorithm

In order to clarify the novel method's methodology, we shall outline the following stages for its application:

Step 1: We use the principle of homotopy to Eq.(10), to get

$$L(f(x)) - f^*(x) + pf^*(x) + p\left(R(f(x)) + N(f(x)) + g(x)\right) = 0. \quad (18)$$

Step 2: Taking the L^{-1} operator to Eq.(18), and we rearrange it to get the following form

$$f(x) = L^{-1}(f^*(x)) - pL^{-1}(f^*(x)) - pL^{-1}\left(R(f(x)) + N(f(x)) + g(x)\right). \quad (19)$$

Step 3: Assume that $f(x) = \sum_{i=0}^{\infty} p^i u_i(x)$ and $f^*(x) = \sum_{i=0}^{\infty} c_i\, C_i(\mathrm{z})$ then, we obtain

$$\boldsymbol{\sum_{i=0}^{\infty} p^i u_i(x) = L^{-1}(\sum_{i=0}^{\infty} c_i\, C_i(\mathrm{z})) - pL^{-1}(\sum_{i=0}^{\infty} c_i\, C_i(\mathrm{z})) - pL^{-1}\left(R(\sum_{i=0}^{\infty} p^i u_i(x)) + N(\sum_{i=0}^{\infty} p^i u_i(x)) + g(x)\right)}. \quad (20)$$

Step 4: By equating the terms on both sides of Eq. (20), which have the same powers p, we obtain:

$$\left.\begin{array}{ll} p^0: & u_0 = L^{-1}(\sum_{i=0}^{\infty} c_i C_i(\mathrm{z})) \\ p^1: & u_1 = -L^{-1}(\sum_{i=0}^{\infty} c_i C_i(\mathrm{z})) - L^{-1}\big(R(u_0) + N(u_0) + g(x)\big) \\ p^2: & u_2 = -L^{-1}\big(R(u_0, u_1) + N(u_0, u_1)\big) \\ p^i: & u_i = -L^{-1}\big(R(u_0, u_1, \dots, u_{i-1}) + N(u_0, u_1, \dots, u_{i-1})\big), \quad i = 3,4, \dots \end{array}\right\}. \quad (21)$$

Step 5: We assume that $u_1 = 0$, and find the values of the unknown coefficient c_i by equating the Chebyshev polynomials and solving the resulting system of equations. Therefore, the analytical solution becomes as follows:

$$\mathrm{f(x) = u_0 = L^{-1}(\sum_{i=0}^{\infty} c_i C_i(z))}\,. \quad (22)$$

3. Mathematical model

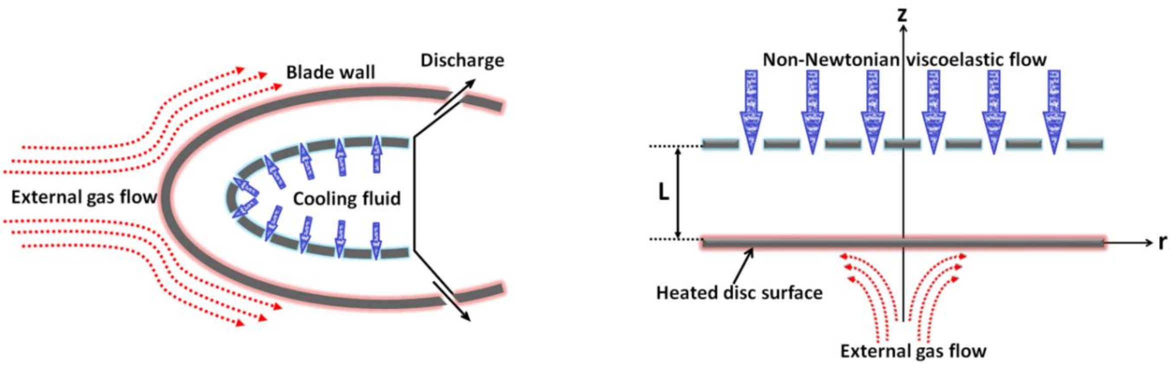


Fig .1. The diagrammatic(right) illustration of the schematic physical problem (left).

3.1 Analysing flow

This research examines the problem of cooling the turbine disc which appears on the right in Fig.1 through the flow of a non-Newtonian viscoelastic fluid where the simultaneous development of flow and heat transfer are investigated. Fig.1(left) represents the illustration of the physical system where the r-axis and z-axis are parallel and normal to the surface of the disc, respectively. At $z = +L$ is the channel's porous disk. The wall corresponding to the r-axis is externally heated because of the passage of gas. To cool the heated wall, the non-Newtonian fluid is uniformly injected into the perforated side from the other wall side. The governing equations of this problem can be written in cylindrical coordinates for a steady, two-dimensional, axisymmetric, and non-Newtonian fluid flow as follows:

$$\frac{\partial(r\,u_r)}{\partial r}+\frac{\partial(r\,u_z)}{\partial z}=0, \tag{23}$$

$$u_r\frac{\partial u_r}{\partial r}+u_z\frac{\partial u_r}{\partial z}=-\frac{1}{\beta}\frac{\partial P}{\partial r}+\frac{1}{\beta}\left[\frac{\partial \mu_{rr}}{\partial r}+\frac{1}{r}(\mu_{rr}-\mu_{\theta\theta})+\frac{\partial \tau_{rz}}{\partial z}\right], \tag{24}$$

$$u_r\frac{\partial u_z}{\partial r}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\beta}\frac{\partial P}{\partial z}+\frac{1}{\beta}\left[\frac{\partial \mu_{zr}}{\partial r}+\frac{1}{r}\mu_{rz}+\frac{\partial \mu_{zz}}{\partial z}\right], \tag{25}$$

with boundary conditions

$$u_r(r,0)=u_z(r,0)=0,\ u_r(r,+L)=0,\ u_z(r,+L)=-V, \tag{26}$$

where u_r is the r-direction velocity, u_z is the z-direction velocity, P is pressure, β is the density, μ_{rr}, μ_{rz}, μ_{zr} and μ_{zz} are components of the stress matrix, V is the injected fluid velocity.

To solve the axisymmetric flow case of the problem shown in Fig.1, it is suitable to clarify a stream function that satisfies the continuity equation as follows:

$$\omega = Vr^2 f_1(x), \tag{27}$$

where $x=\frac{z}{L}$ and the velocity components in z-direction and r-direction are defined as

$$u_r=\frac{Vr}{L}f_1'(x),\ u_z=-2Vf_1(x). \tag{28}$$

Via using Eq.(27,28) the equations of momentum become [7]:

$$f_1''''+2Ref_1\,f_1'''-k\,Re(4f_1''f_1'''+2f_1'f_1'''')=0, \tag{29}$$

with the boundary conditions

$$f_1(0)=f_1'(0)=f_1'(1)=0,\ f_1(1)=1, \tag{30}$$

where k is the injection Reynolds number and Re is the Reynolds number.

3.2 Analysing heat transfer

The dimensionless energy equation with the viscous squandering in the current problem is as follows [7]:

$$\rho\, C\left(u_r\frac{\partial T}{\partial r}+u_r\frac{\partial T}{\partial r}\right)=\dot{k}\,\nabla^2 T+\emptyset, \tag{31}$$

$$\emptyset=\mu_{rr}\frac{\partial u_r}{\partial r}+\mu_{\theta\theta}\frac{u_r}{r}+\mu_{zz}\frac{\partial u_z}{\partial z}+\mu_{rz}\left(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right), \tag{32}$$

where $C, T, \dot{k},$ and $\emptyset$ are the specific heat, temperature, fluid coefficient, and dissipation function, respectively. By ignoring the influence of squandering, we obtain the non-dimensional equation:

$$f_2''-Pr\,Re(n\,f_1'\,f_2+f_1f_2')=0,\quad n=0,2,3,4,\ldots, \tag{33}$$

with boundary conditions

$$f_2(0)=1,\ f_2(1)=0, \tag{34}$$

where Pr is the Prandtl number and n is the power-law index.

4. The CHPM's application to the mathematical model

The CHPM technique was used to solve the current boundary value problem in equations (29, 30, 33, and 34). The primary phases of the new technique are given below:

Step 1: Applying the homotopy property on Eqs.(29,33), we get,

$$f_1''''-f_1^*+pf_1^*+p\big(2Ref_1\,f_1'''-k\,Re(4f_1''f_1'''+2f_1'f_1'''')\big)=0, \tag{35}$$

$$f_2''-f_2^*+pf_2^*-p\big(Pr\,Re(n\,f_1'\,f_2+f_1f_2')\big)=0, \tag{36}$$

Step 2: Taking $L_1^{-1} = \int_0^x \int_0^x \int_0^x \int_0^x (.)\, dx\, dx\, dx\, dx$ and $L_2^{-1} = \int_0^x \int_0^x (.)\, dx\, dx$ for both sides of Eqs.(35,36) respectively yields

$$f_1 = f_1(0) + x f_1'(0) + \frac{x^2}{2} f_1''(0) + \frac{x^3}{6} f_1'''(0) + L_1^{-1}(f_1^*) - p\, L_1^{-1}(f_1^*) - p\, L_1^{-1}\big(2Re f_1\, f_1''' - k\, Re(4 f_1'' f_1''' + 2 f_1' f_1'''')\big), \quad (37)$$

$$f_2 = f_2(0) + x f_2'(0) + L_2^{-1}(f_2^*) - p L_2^{-1}(f_2^*) + p L_2^{-1}\big(Pr\, Re(n\, f_1'\, f_2 + f_1 f_2')\big), \quad (38)$$

Step 3: Assuming that $f_1(x) = \sum_{i=0}^{\infty} p^i u_i(x)$, $f_2(x) = \sum_{i=0}^{\infty} p^i v_i(x)$, $f_1^*(x) = \sum_{i=0}^{m_1} c_i\, T_i^*(x)$ and $f_2^*(x) = \sum_{i=0}^{m_2} d_i\, T_i^*(x)$ then, we get

$$\sum_{i=0}^{\infty} p^i u_i = \frac{x^2}{2} \mathrm{A}_1 + \frac{x^3}{6} \mathrm{A}_2 + L_1^{-1}\Big(\sum_{i=0}^{m_1} c_i\, T_i^*\Big) - p\, L_1^{-1}\Big(\sum_{i=0}^{m_1} c_i\, T_i^*\Big) - p\, L_1^{-1}\Big(2Re \sum_{i=0}^{\infty} p^i u_i \sum_{i=0}^{\infty} p^i u'''_i - k\, Re\big(4 \sum_{i=0}^{\infty} p^i u''_i \sum_{i=0}^{\infty} p^i u'''_i + 2 \sum_{i=0}^{\infty} p^i u'_i \sum_{i=0}^{\infty} p^i u''''_i\big)\Big), \quad (39)$$

$$\sum_{i=0}^{\infty} p^i v_i = 1 + x\, B + L_2^{-1}\Big(\sum_{i=0}^{m_2} d_i\, T_i^*\Big) - p L_2^{-1}\Big(\sum_{i=0}^{m_2} d_i\, T_i^*\Big) + p L_2^{-1}\Big(Pr\, Re\big(n \sum_{i=0}^{\infty} p^i u'_i \sum_{i=0}^{\infty} p^i v_i + \sum_{i=0}^{\infty} p^i u_i \sum_{i=0}^{\infty} p^i v'_i\big)\Big), \quad (40)$$

where $\mathrm{A}_1 = f_1''(0)$, $\mathrm{A}_2 = f_1'''(0)$ and $\mathrm{B} = f_2'(0)$

Step 4: equaling the terms for Eqs.(39,40) which have the same powers of p

$$p^0: \begin{cases} u_0 = \frac{x^2}{2} \mathrm{A}_1 + \frac{x^3}{6} \mathrm{A}_2 + L_1^{-1}\big(\sum_{i=0}^{m_1} c_i\, T_i^*\big) \\ v_0 = 1 + x\, B + L_2^{-1}\big(\sum_{i=0}^{m_2} d_i\, T_i^*\big) \end{cases}, \quad (41)$$

$$p^1: \begin{cases} u_1 = -\, L_1^{-1}\big(\sum_{i=0}^{m_1} c_i\, T_i^*\big) - L_1^{-1}\big(2Re\, u_0\, u_0''' - k\, Re(4u_0''\, u_0''' + 2u_0' u_0'''')\big) \\ v_1 = -L_2^{-1}\big(\sum_{i=0}^{m_2} d_i\, T_i^*\big) + L_2^{-1}\big(Pr\, Re(n\, u'_0\, v_0 + u_0\, v'_0)\big) \end{cases}, \quad (42)$$

and so on.

Step 5: We exchange x, x^2 and x^3 in terms of the Chebyshev polynomials, and then the values of $(\mathrm{A}_1, \mathrm{A}_2)$ and B are found by using the boundary conditions ($f_1'(1) = 0$, $f_1'(1) = 1$) and $f_2(1) = 0$ respectively.

Step 6: We find the values c_i by assuming that $u_1 = 0$. Therefore, the analytical approximate solution of f_1 becomes as follows**:**

$$f_1 = u_0 = \frac{x^2}{2} \mathrm{A}_1 + \frac{x^3}{6} \mathrm{A}_2 + L_1^{-1}\Big(\sum_{i=0}^{m_1} c_i\, T_i^*\Big), \quad (43)$$

and similarly, we find the values of d_i and thus we get the analytical approximate solution of f_2 as follows**:**

$$f_2 = v_0 = 1 + x\, B + L_2^{-1}\Big(\sum_{i=0}^{m_2} d_i\, T_i^*\Big), \quad (44)$$

It should be noted that in steps 5 and 6, the relations(Derivation, Multiplication, Integration) that were mentioned earlier are applied, also, in step 6, the values of c_i and d_i are found by solving a system of algebraic equations that is constructed depending on T_i^* coefficients.

5. Results and discussion

The CHPM was used to solve the problem of the turbine disc cooling through a non-Newtonian fluid flow in the porous wall of the axisymmetric channel Fig. 1. The validity of the results obtained using this new method has been proven by comparing them with the results of the numerical methods. Fig. 2 shows a good agreement with the Runge-Kutta method with regard to the velocity f_1 and temperature distribution f_2 for different values of the effective parameters.

The Tables(2-5) prove the high accuracy of the results obtained using this method through the smallness of the percentage error $\left(E_{\%}(x) = \left|\frac{f_{NM}(x) - f_{CHPM}(X)}{f_{NM}(x)}\right| \times 100\right)$ and the average percentage error $\left(E_{\text{ave}} = \frac{1}{11} \sum_{i=0}^{10} E_{\%}\left(\frac{i}{10}\right)\right)$ of the velocity and heat distribution compared to other methods. In all these tables only 1 iteration of CHPM was used, while 34 iterations in LeNN-GNDO-SQP, 10 iterations in DTM, 5 neurons in HNNPSO and 2 iterations in (YTHPM, HPM, VIM, ADM, HAM, OHAM). The agreement of the analytical results with the numerical results in Fig.2 and the smallness of errors in Tables 1- 4 show the efficiency of CHPM and enhances confidence in the validity of these results. From Figs.3 and 4, we can see that the Reynolds number(Re) and cross viscosity parameter(k) have the same effect on velocity since increasing them causes an increase velocity in the z-direction and the maximum velocity in the r-direction tends to the warm plate ($z = 0$). Fig.5 shows the effect of Reynolds number(Re), cross viscosity parameter(k), Prandtl number(pr), and power law index(n) on the temperature distribution. This figure shows that as they rise, the temperature distribution decreases.

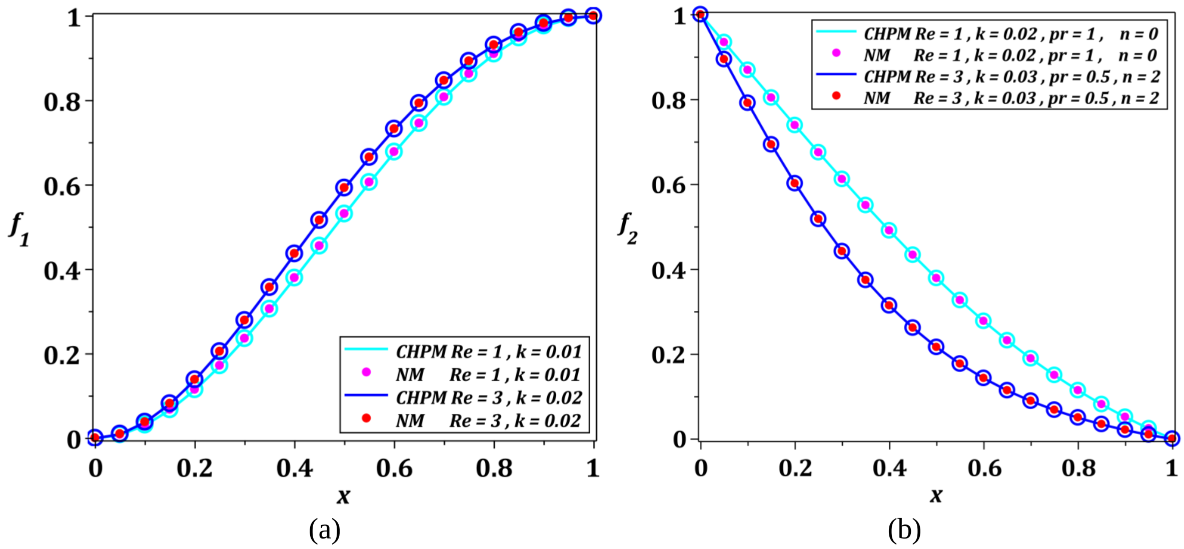


Fig. 2. Approximate solutions of CHPM Compared to numerical method (NM) for (a) f_1 and (b) f_2.

Table 2. A comparison between the percentage error of CHPM and another method for f_1 when $Re = 0.5$ and $k = 0.01$.

x	CHPM	LeNN-GNDO-SQP[33]	HNNPSO[34]	DTM[2]	VIM[6]	HAM[6]
0	0	0	0	0	0	0
0.1	0	8.52E-4	2.23E-1	2.57E-1	1.63E-2	3.38E-6
0.2	0	6.85E-4	1.22E-1	1.56E-1	6.59E-2	3.66E-6
0.3	0	7.75E-5	4.55E-2	7.66E-2	1.40E-1	4.43E-6
0.4	0	5.20E-5	6.50E-3	2.00E-2	2.21E-1	5.47E-6
0.5	2.13E-7	4.25E-5	1.04E-2	1.49E-2	2.84E-1	5.43E-6
0.6	0	5.59E-6	4.63E-2	3.18E-2	3.05E-1	3.92E-6
0.7	0	2.84E-5	4.12E-2	3.35E-2	2.62E-1	2.01E-6
0.8	1.14E-7	8.75E-6	2.65E-2	2.44E-2	1.40E-1	4.43E-7
0.9	0	4.65E-7	9.00E-3	9.65E-3	7.19E-2	0
1	0	5.62E-10	0	0	3.81E-1	0

Table 3.a. A comparison between the average error of CHPM and another method for f_1.

Re	k	CHPM	LeNN-GNDO-SQP[33]	YTHPM [7]	HNNPSO [34]	HPM [6]	VIM [6,35]
0.1	0.01	2.59E-04	-	3.72E-04	-	-	-
0.5	0.001	7.21E-08	-	-	-	2.57E-03	4.09E-03
0.5	0.01	2.98E-08	1.59E-04	1.40E-05	4.82E-02	-	1.72E-01
0.5	0.1	1.21E-02	-	-	-	-	2.43E-01
1	0.001	1.67E-06	-	-	-	3.77E-02	4.26E-01
1	0.01	3.11E-06	7.37E-03	-	-	8.11E-02	3.38E-01
1	0.1	3.83E-04	-	-	-	-	-
1.5	0.01	5.73E-03	5.19E-02	-	-	-	-
2	0.01	7.42E-03	2.98E-01	-	-	-	-

Table 3.b. A comparison between the average error of CHPM and another method for f_1.

Re	k	CHPM	GM[5]	DTM[2]	ADM[1]	OHAM[36]	HAM[37]
0.1	0.01	2.59E-04	-	-		-	-
0.5	0.001	7.21E-08	3.54E-03	-		-	-
0.5	0.01	2.98E-08	-	5.68E-02		-	2.61E-06
1	0.01	3.11E-06	-	-	3.14E-06	-	-
1	0.1	3.83E-04	-	-	-	1.08E-03	1.84E-03

Table 4. A comparison between the percentage error of CHPM and another method for f_2 when $Re = 0.5$, $k = 0.01$, $pr = 1$ and $n = 0$.

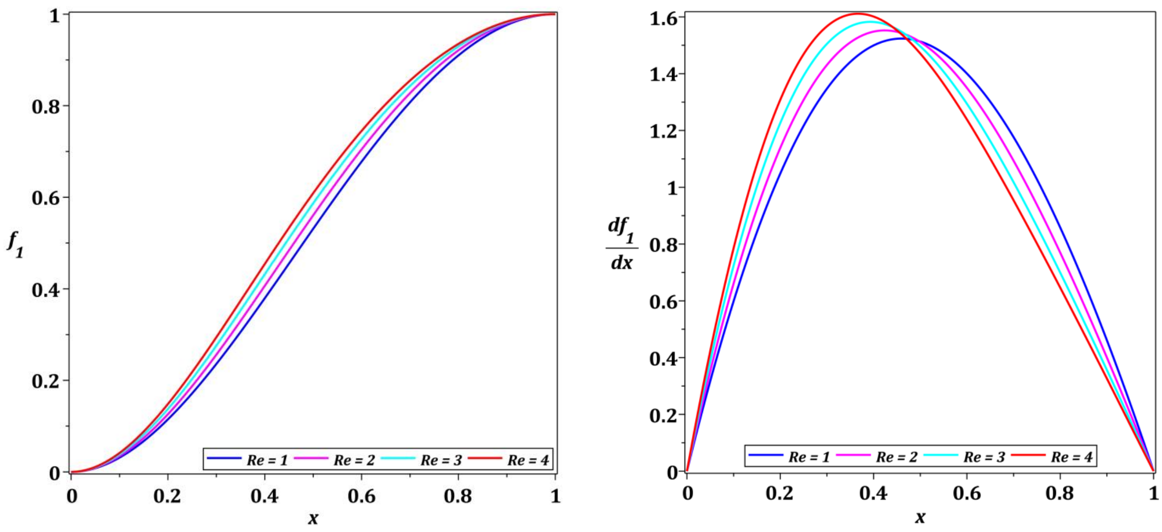
x	CHPM	HNNPSO[34]	VIM[6]	HAM[6]
0	0	0.00E+00	0.00E+00	0.00E+00
0.1	1.66E-07	2.00E-04	7.51E+00	1.24E-06
0.2	0	1.00E-04	1.32E+01	3.64E-06
0.3	6.13E-07	8.00E-04	1.75E+01	8.53E-06
0.4	1.14E-06	2.00E-03	2.07E+01	1.61E-05
0.5	4.91E-07	3.40E-03	2.30E+01	2.08E-05
0.6	1.28E-06	4.70E-03	2.48E+01	1.22E-05
0.7	1.88E-06	5.80E-03	2.66E+01	1.74E-05
0.8	1.16E-06	6.50E-03	2.84E+01	6.46E-05
0.9	0	9.60E-03	3.06E+01	1.10E-04
1	0	0.00E+00	3.33E+01	0.00E+00

Table 5.a. A comparison between the average error of CHPM and another method for f_2.

Re	k	p	n	CHPM	YTHPM[7]	HNNPS[34]	HPM[6]	VIM[6,3]
0.1	0.01	0.7	0	5.38E-05	2.20E-03	-	-	-
0.5	0.001	1	2	4.83E-06	-	-	-	-
0.5	0.01	0.5	0	1.27E-06	1.43E-03	-	-	-
0.5	0.01	1	0	6.12E-07	6.50E-04	3.01E-03	-	2.05E+01
1	0.01	0.5	0	1.44E-05	-	-	8.10E-02	7.47E-01
1	0.01	0.5	2	1.32E-05	-	-	6.64E-02	8.56E-01
1	0.02	1	0	4.85E-06	-	-	-	-
1	0.1	1	2	1.76E-04	-	-	-	-

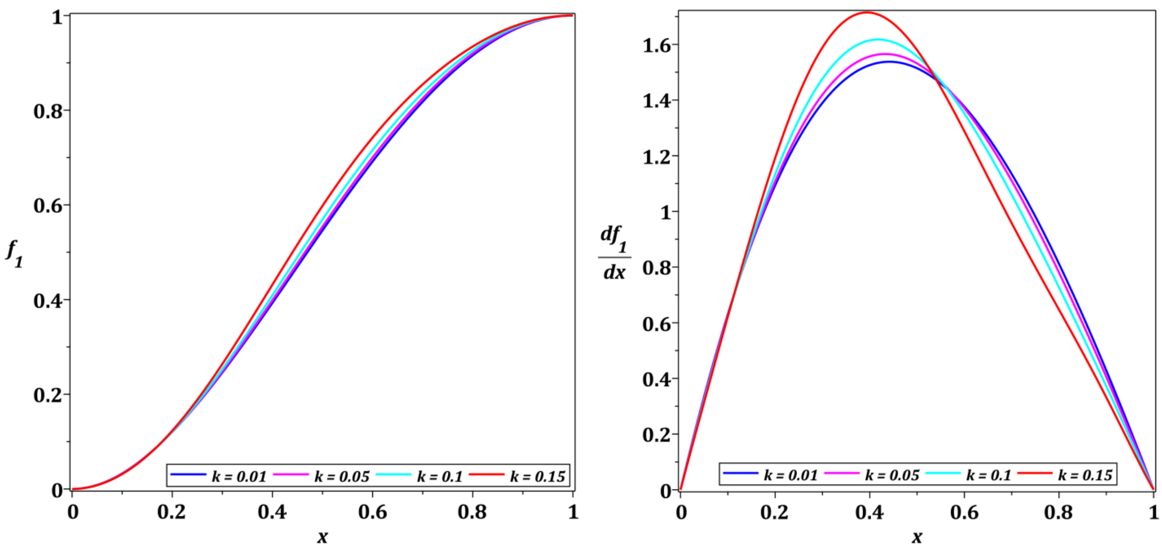
Table 5.b. A comparison between the average error of CHPM and another method for f_2.

Re	k	p	n	CHPM	GM[5]	ADM[1]	OHAM[3]	HAM[37]
0.1	0.01	0.7	0	5.38E-05	-	-	-	-
0.5	0.001	1	2	4.83E-06	1.69E-01	-	-	-
0.5	0.01	0.5	0	1.27E-06	-	-	-	7.20E-06
0.5	0.01	1	0	6.12E-07	-	-	-	2.31E-05
1	0.01	0.5	0	1.44E-05	-	-	-	-
1	0.01	0.5	2	1.32E-05	-	-	-	-
1	0.02	1	0	4.85E-06	-	8.12E-06	-	-
1	0.1	1	2	1.76E-04	-	-	2.66E-03	1.39E-03



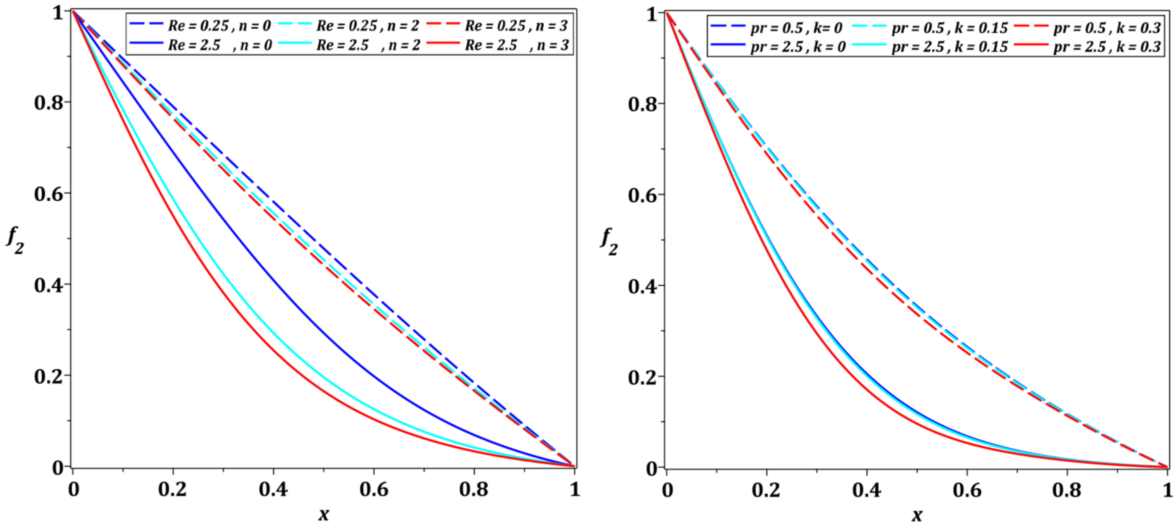
(a) (b)

Fig .3. The impact of the Reynolds number on the z-direction(a) and r-direction(b) velocity components for $k = 0.01$.



(a) (b)

Fig .4. The impact of the injection Reynolds number on the z-direction(a) and r-direction(b) velocity components for Re = 1.5.



(a) (b)

Fig .5. The impact of the Re, n, pr and k on the temperature distribution components for (a) $k = 0.01$, $pr = 0.7$ and (b) $Re = 1$, $n = 3$.

6. Convergent analysis of CHPM

In this section, we will examine the convergent analysis of the approximate solution achieved by CHPM. Consider the system of nonlinear equations (29,33) in the following form:

$$\left.\begin{aligned} f_1(x) &= G_1\big(f_1(x),\ f_2(x)\big) \\ f_2(x) &= G_2\big(f_1(x),\ f_2(x)\big) \end{aligned}\right\}, \tag{45}$$

where G_1 and G_2 are non-linear operators. The solution by the present approach is equivalent to the following sequence:

$$\left.\begin{aligned} S_{n1} &= \textstyle\sum_{i=0}^{n1} u_i = \sum_{i=0}^{n1} c_i\, T_i^* \\ K_{n2} &= \textstyle\sum_{i=0}^{n2} v_i = \sum_{i=0}^{n2} d_i\, T_i^* \end{aligned}\right\}, \tag{46}$$

Table 6. Convergence of analytical approximate solutions for f_1

Re	k	α^1	α^2	α^3	α^4	α^5
1	0.01	6.42E-01	8.73E-02	2.08E-02	5.58E-03	5.84E-04
2	0.01	6.29E-01	8.54E-02	3.95E-02	1.09E-02	4.92E-04
3	0.01	6.17E-01	8.22E-02	5.54E-02	1.56E-02	2.29E-04
4	0.01	6.06E-01	7.83E-02	6.85E-02	1.94E-02	1.41E-03
1.5	0.01	6.35E-01	8.65E-02	3.05E-02	8.32E-03	6.23E-04
1.5	0.05	6.34E-01	8.83E-02	3.52E-02	1.03E-02	1.37E-03
1.5	0.1	6.30E-01	9.06E-02	4.38E-02	1.42E-02	2.47E-03
1.5	0.15	6.22E-01	9.22E-02	5.94E-02	2.21E-02	3.31E-03

Table 7. convergence of analytical approximate solutions for f_2

Re	k	pr	n	α^1	α^2	α^3	α^4	α^5
0.25	0.01	0.7	0	5.99E-01	1.54E-02	3.20E-03	1.20E-04	1.11E-04
0.25	0.01	0.7	2	6.11E-01	3.35E-02	1.03E-03	3.97E-04	2.22E-04
0.25	0.01	0.7	3	6.17E-01	4.23E-02	1.45E-03	1.02E-03	3.89E-04
2.5	0.01	0.7	0	7.34E-01	1.76E-01	1.19E-02	1.07E-02	3.48E-04
2.5	0.01	0.7	2	8.00E-01	2.97E-01	3.77E-02	9.86E-03	5.35E-03
2.5	0.01	0.7	3	8.24E-01	3.45E-01	5.97E-02	7.61E-03	7.51E-03
1	0	0.5	3	6.74E-01	1.17E-01	7.15E-03	3.96E-03	1.43E-03
1	0.15	0.5	3	6.76E-01	1.19E-01	7.80E-03	4.38E-03	1.64E-03
1	0.3	0.5	3	6.83E-01	1.33E-01	1.43E-02	5.12E-03	3.20E-03
1	0	2.5	3	8.67E-01	4.17E-01	8.30E-02	1.02E-02	8.74E-03
1	0.15	2.5	3	8.70E-01	4.23E-01	8.55E-02	1.11E-02	9.78E-03
1	0.3	2.5	3	8.89E-01	4.65E-01	1.11E-01	1.69E-02	8.61E-03

Theorem 6.1 (Convergence of the problem of the turbine disc cooling)
Let G_1 and G_2 are the operators from a Hilbert space $\mathcal{H}$ into $\mathcal{H}$ and $f_1\,(x)$ and $f_2\,(x)$ are the analytical solution of equations (29, 33). The approximate solutions $\sum_{i=0}^{n1} u_i = \sum_{i=0}^{n1} c_i\, T_i^*$ and $\sum_{i=0}^{n2} v_i = \sum_{i=0}^{n2} d_i\, T_i^*$ are convergent to analytical solutions $f_1\,(x),\ f_2\,(x)$ respectively when $\exists\ 0 \leq \alpha < 1$, $\|u_{i+1}\| \leq\ \alpha\,\|u_i\|\ \forall\ i \in \mathbb{N} \cup \{0\}$ and $\exists\ 0 \leq \gamma < 1$, $\|v_{i+1}\| \leq\ \gamma\|v_i\|\ ,\forall\ i \in \mathbb{N} \cup \{0\}$.

Proof: Firstly, we will prove that for $f_1\,(x)$,
we want to show that $\{S_{n1}\}_{n1=0}^{\infty}$ is a Cauchy sequence,

$$\|S_{n1+1} - S_{n1}\| = \|u_{n1+1}\| \leq \alpha\|u_{n1}\| \leq \alpha^2\|u_{n1-1}\| \leq \cdots \leq\ \alpha^{n1}\|u_1\| \leq \alpha^{n1+1}\|u_0\|. \quad \textbf{(47)}$$

Now for $n1, m1 \in \mathbb{N}\ , n1 \geq m1$

$$\begin{aligned}\|S_{n1} - S_{m1}\| &= \|(S_{n1} - S_{n1-1}) + (S_{n1-1} - S_{n1-2}) + \cdots + (S_{m1+1} - S_{m1})\| \\ &\leq \|S_{n1} - S_{n1-1}\| + \|S_{n1-1} - S_{n1-2}\| + \cdots + \|S_{m1+1} - S_{m1}\| \\ &\leq \alpha^{n1}\|u_0\| + \alpha^{n1-1}\|u_0\| + \cdots + \alpha^{m+1}\|u_0\| \\ &\leq (\alpha^{m1+1} + \alpha^{m1+2} + \cdots + \alpha^{n1})\|u_0\| = \alpha^{m1+1}\frac{1-\alpha^{n1-m1}}{1-\alpha}\|u_0\| \qquad (48)\end{aligned}$$

Hence, $\lim_{n1,m1\to\infty}\|S_{n1} - S_{m1}\| = 0$, it means that $\{S_{n1}\}_{n1=0}^{\infty}$ is a Cauchy sequence in the Hilbert space $\mathcal{H}$, therefore there exists $S \in \mathcal{H}$ such that $\lim_{n1\to\infty} S_{n1} = S$, where $S = f_1\,(x)$.

The proof of f_2 is done in the same way. ■

Established on theorem 6.1, the parameter values α^{n1} must be calculated to achieve convergence using the following relationship:

$$\alpha^{n1} \geq \begin{cases} \dfrac{\|u_{n1}\|}{\|u_0\|}, & \|u_0\| \neq 0, \quad n1 = 2,3,4,\dots \\ 0 \quad , & otherwise \end{cases}$$

where $\alpha = \frac{\|u_1\|}{\|u_0\|} < 1$.

We can now determine the convergence of the analytical approximate solutions using this relationship, as Tables 6 and 7 show.

7. Conclusions

In this paper, a novel approach (CHPM) is successfully applied to provide an approximate analytical solution for the turbine disc cooling problem. The effects of several parameters, including the injection Reynolds number (Re), the cross viscosity parameter (k), the Prandtl number (Pr), and the power law index (n), on velocity and temperature distribution are investigated. From the obtained results, the following conclusions are drawn:

- Increasing the Reynolds number leads to an increase in the velocity value with an increase in the curvature of the temperature distribution and a decrease in its value.
- The velocity reaches its highest value in the channel's centre with low Reynolds numbers.
- Increasing the cross viscosity leads to an increase in the velocity value, but a decrease in the temperature distribution.
- Increasing the Prandtl number and the power law index leads to a decrease in the temperature distribution.
- CHPM has a good convergence as shown in Tables(6,7) and a high accuracy compared to other methods as clear in Tables(2-5).

The results obtained affirm the validity of the new method, which is characterized by reduced iteration, agreement with previous studies, minimal errors, and excellent convergence. As a result, it can be effectively utilized to investigate more complex fluid flow problems and other application model problems that hold significant real-world relevance.

8. References

[1] A.S. Dogonchi, D.D. Ganji, "Investigation of heat transfer for cooling turbine disks with a non-Newtonian fluid flow using DRA," Case Stud. Therm. Eng., vol. 6, pp. 40-51, 2015.Doi:https://doi.org/10.1016/j.csite.2015.06.002

[2] S. Sepasgozar, M. Faraji, P. Valipour, "Application of differential transformation method (DTM) for heat and mass transfer in a porous channel," Propuls. Power Res., vol. 6, pp. 41-48, 2017.Doi:https://doi.org/10.1016/j.jppr.2017.01.001

[3] H. Mirgolbabaee, S.T. Ledari, M. Sheikholeslami, D.D. Ganji, "Semi-analytical investigation of momentum and heat transfer of a non-Newtonian fluid flow for specific turbine cooling application using AGM," Int. J. Appl. Comput. Math. 3(Suppl 1), S1463-S1475, 2017.Doi:https://doi.org/10.1007/s40819-017-0416-3

[4] N. Singh, R. Yadav, "Investigation of heat transfer of non-Newtonian fluid in the presence of a porous wall," Int. J. Eng. Technol. Manag. Res. 4, pp. 74–92, 2017. Doi:https://doi.org/10.5281/zenodo.1140081

[5] G.A. Sheikhzadeh, M. Mollamahdi, M. Abbaszadeh, "Analytical study of flow field and heat transfer of a non-Newtonian fluid in an axisymmetric channel with a permeable wall," J. Comput. Appl. Res. 7, pp. 161–173, 2018.Doi:https://doi.org/10.22061/jcarme.2017.2003.1174

[6] A.T. Akinshilo, M. Sanusi, M.G. Sobamowo, A.E. Olorunnisola, "Thermal performance analysis of non‐Newtonian fluid transport through turbine discs," Heat Transfer, vol. 51, no. 1, pp. 451-469, Jan 2021.Doi:https://doi.org/10.1002/htj.22315

[7] T.A.J. Al-Griffi, A.-S.J. Al-Saif, "Yang transform-homotopy perturbation method for solving a non-Newtonian viscoelastic fluid flow on the turbine disk," Z. Angew. Math. Mech., e202100116, 2022.Doi:https://doi.org/10.1002/zamm.202100116

[8] A.S.J. Al-Saif, M.S. Abdul-Wahab, "A new technique for simulation the Zakharov-Kuznetsov equation," J. of Adv. Math., vol. 14, no. 2, pp. 7912-7920, 2018.Doi:https://doi.org/10.24297/jam.v14i2.7559

[9] A.S.J. Al-Saif, M.S. Abdul-Wahab, "Application of new simulation scheme for the nonlinear biological population model," Num. Com. Meth. Sci. Eng., vol. 1, no. 2, pp. 89-99, 2019.Doi:http://dx.doi.org/10.18576/ncmsel/010204

[10] Umesh, M. Kumar, "Approximate solution of singular IVPs of Lane–Emden type and error estimation via advanced Adomian decomposition method," J. Appl. Math. Comput., vol. 66, pp. 527–542, 2021.Doi:https://doi.org/10.1007/s12190-020-01444-2

[11] A.S.J. Al-Saif, T.A.J. Al-Griffi, "Analytical simulation for transient natural convection in a horizontal cylindrical concentric annulus," J. Appl. Comput. Mech., vol. 7, no. 2, pp. 621-637, 2021.Doi:https://doi.org/10.22055/JACM.2020.35278.2617

[12] T.A.J. Al-Griffi, A.S.J. Al-Saif, "Akbari-Ganji homotopy perturbation method for analyzing the pulsatile blood flow in tapered stenosis arteries under the effect of magnetic field together with the impact of mass and heat transfer," J. Comput. Appl. Mech., vol. 53, no. 4, pp. 543-570, 2022.Doi:https://doi.org/10.22059/JCAMECH.2022.348399.757

[13] Y.A. Abdulameer, A.S.J. Al-Saif, "A well-founded analytical technique to solve 2D viscous flow between slowly expanding or contracting walls with weak permeability," J. Adv. Res. Fluid Mech. Therm. Sci., vol. 97, no. 2, pp. 39-56, 2022.Doi:https://doi.org/10.37934/arfmts.97.2.3956

[14] A.K. Al-Jaberi1, M.S. Abdul-Wahab, R.H. Buti, "A new approximate method for solving linear and nonlinear differential equation systems," AIP Conf. Proc., vol. 2398, no. 1, p. 060082, 2022.Doi:https://doi.org/10.1063/5.0094138

[15] H. Aminikhah, M. Hemmatnezhad, "An efficient method for quadratic Riccati differential equation," Commun. Nonlinear Sci. Numer. Simulat., vol. 15, pp. 835–839, 2010.Doi:https://doi.org/10.1016/j.cnsns.2009.05.009

[16] M.R. Gad-Allah, T.M. Elzaki, "Application of new homotopy perturbation method for solving partial differential equations," J. Comput. Theor. Nanosci., vol. 15, no. 2, pp. 500-508, 2018.Doi:https://doi.org/10.1166/jctn.2018.6725

[17] D.K. Maurya, R. Singh, Y.K. Rajoria, "A mathematical model to solve the Burgers-Huxley equation by using new homotopy perturbation method," Int. J. Math., Eng. and Manage. Sci., vol. 4, no. 6, pp. 1483-1495, 2019.Doi:https://dx.doi.org/10.33889/IJMEMS.2019.4.6-117

[18] R. Kumar, A.K. Singh, S.S. Yadav, "New homotopy perturbation method for analytical solution of telegraph equation," Turk. J. Comput. Math. Educ. (TURCOMAT), vol. 12, no. 12, pp. 2144-2155, 2021.Doi:https://doi.org/10.17762/turcomat.v12i12.7760

[19] B. Seethalakshmi, V. Ananthaswamy, S. Narmatha, "Application of new homotopy perturbation method in solving a simple predator prey model with rich dynamics," Adv. and Appl. Math. Sci., vol. 21, no. 4, pp. 2015-2025, 2022.

[20] K. Pal, V. G. Gupta, H. Singh, V. Pawar, "Enlightenment Of Heat Diffusion Using New Homotopy Perturbation Method," J. Appl. Sci. Eng., vol. 27, no. 3, pp. 2213-2216,2023.Doi:http://dx.doi.org/10.6180/jase.202403_27(3).0007

[21] J. Wu, Y. Zhang, L. Chen, Z. Luo, "A Chebyshev interval method for nonlinear dynamic systems under uncertainty," App. Math. Modell., vol. 37, no. 6, pp. 4578-4591, 2013.Doi:https://doi.org/10.1016/j.apm.2012.09.073

[22] Y. M. Hamada, "A new accurate numerical method based on shifted Chebyshev series for nuclear reactor dynamical systems," Sci. Technol. Nucl. Install, vol. 15, ID7105245, 2018.Doi:https://doi.org/10.1155/2018/7105245

[23] O. B. Arushanyan, S. F. Zaletkin, "On some analytic method for approximate solution of systems of second order ordinary differential equations," Moscow Univ. Math. Bull, vol. 74, no. 3, pp. 127-130, 2019.Doi:https://doi.org/10.3103/S0027132219030057

[24] K. K. Ali, M. A. Abd El Salam, E. M. Mohamed, B. Samet, S. Kumar, M. S. Osman, "Numerical solution for generalized nonlinear fractional integro-differential equations with linear functional arguments using Chebyshev series," Adv. Difference Equations, vol. 2020, no. 1, pp. 1-23, 2020.Doi:https://doi.org/10.1186/s13662-020-02951-z
[25] F. Wang, Q. Zhao, Z. Chen, C. M. Fan, "Localized Chebyshev collocation method for solving elliptic partial differential equations in arbitrary 2D domains," Appl. Math. Comput., vol. 397, pp. 1-13, 2021.Doi:https://doi.org/10.1016/j.amc.2020.125903
[26] S. F. Zaletkin, "Approximate integration of ordinary differential equations using Chebyshev series with precision control," Matem. Mod., vol. 34, no. 6, pp. 53-74, 2022.Doi:https://doi.org/10.20948/mm-2022-06-04
[27] M. Izadi, Ş. Yüzbaşı, D. Baleanu, "Taylor–Chebyshev approximation technique to solve the 1D and 2D nonlinear Burgers equations," Math. Sci., vol. 16, no. 4, pp. 459-471, 2022.Doi:https://doi.org/10.1007/s40096-021-00433-1
[28] S. F. Zaletkin, "Approximate integration of ordinary differential equations using the Chebyshev series with precision control," Math. Models Comput. Simul., vol. 15, pp. 34–46, 2023.Doi:https://doi.org/10.1134/S2070048223010155
[29] J. Duan, L. Jing, "The solution of the time-space fractional diffusion equation based on the Chebyshev collocation method," Indian J. Pure Appl. Math., 2023.Doi:https://doi.org/10.1007/s13226-023-00495-y
[30] J. C. Mason, D. C. Handscomb, "Chebyshev Polynomials," Chapman and Hall/CRC, 2002.Doi:https://doi.org/10.1201/9781420036114
[31] M. H. Mudde, "Chebyshev Approximation," University of Groningen, Netherlands, Faculty of Science and Engineering, 2017.
[32] H. C. Thacher Jr, "Conversion of a power to a series of Chebyshev polynomials," Commun. ACM, vol. 7, no. 3, pp. 181-182, 1964.Doi:https://doi.org/10.1145/363958.363998
[33] N. A. Khan, M. Sulaiman, P. Kumam, F. K. Alarfaj, "Application of Legendre polynomials based neural networks for the analysis of heat and mass transfer of a non-Newtonian fluid in a porous channel," Adv. Continuous Discrete Mod., vol. 2022, no. 1, pp. 1-32, 2022.Doi:https://doi.org/10.1186/s13662-022-03676-x
[34] R. Mirzaei, M. Ghalambaz, A. Noghrehabadi, "Study of the flow and heat transfer of a viscoelastic fluid using hybrid neural network-particle swarm optimization (HNNPSO)," J. Therm Eng., vol. 7, no. 4, pp. 791-805, 2021.Doi:https://doi.org/10.18186/thermal.929636
[35] F. Shakeri, A. Abbasi, M. Naeimaei, A. Yekrangi, A. Kolahdooz, "Variational iteration method for the heat transfer of a Non-Newtonian fluid flow in an axisymmetric channel with a porous wall," World Appl. Sci. J., vol. 16, pp. 26-30, 2012.
[36] F. Mabood, W. A. Khan, A. I. Ismail, "Optimal homotopy asymptotic method for flow and heat transfer of a viscoelastic fluid in an axisymmetric channel with a porous wall," PLoS One, vol. 8, no. 12, pp. 1-8, 2013.Doi:https://doi.org/10.1371/journal.pone.0083581
[37] M. Esmaeilpour, G. Domairry, N. Sadoughi, A. G. Davodi, "Homotopy analysis method for the heat transfer of a non-Newtonian fluid flow in an axisymmetric channel with a porous wall," Commun. Nonlinear Sci. Numer. Simulat., vol. 15, no. 9, pp. 2424-2430, 2010.Doi:https://doi.org/10.1016/j.cnsns.2009.10.004

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طريقة تشيبيشيف-الاضطراب المتماثل لدراسة التدفق وانتقال الحرارة لتدفق السوائل غير النيوتونية على قرص توربيني

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الملخص

تم في هذا البحث تقديم طريقة جديدة لدراسة تأثير المائع غير النيوتوني على الجريان وتوزيع درجة الحرارة عند تبريد قرص التوربين. تعتمد الطريقة الجديدة على طريقة الاضطراب المثلي التي تم تطويرها باستخدام سلسلة تشيبيشيف. وتمت مقارنة نتائج الطريقة المقترحة مع النتائج التي تم الحصول عليها باستخدام الطرق العددية في الأدبيات السابقة للتأكد من صدق الطريقة، حيث أظهرت توافقا جيدا. تم استكشاف تأثير العديد من العوامل الفيزيائية على سرعة التدفق وانتشار درجة الحرارة، مثل رقم رينولدز، ومعلمة اللزوجة المتقاطعة، ورقم براندتل، وقانون الطاقة. وكانت النتائج التي تم الحصول عليها باستخدام الطريقة المقترحة أكثر دقة من الطرق الأخرى المستخدمة لحل المشكلة الحالية. علاوة على ذلك، توضح الأشكال وجداول الأخطاء فعالية الطريقة الجديدة وكفاءتها.

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