

Spreads on Three-dimensional Projective Space order (19)

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A R T I C L E I N F O		A B S T R A C T
Received	08 November 2025	The aim of this work is to study the three-dimensional projective space $PG(3, 19)$ by using algebraic equations. We determine the points, lines, and planes of this space. In addition, we construct a (K, L) -spread, which is defined as a set of K mutually disjoint lines in $PG(3, 19)$. We prove that the maximum complete (K, L) -spread in $PG(3, 19)$ consists of 362 lines, and that this spread covers all points of the space, hence forming a span.
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1. Introduction

The study of three-dimensional projective spaces $PG(3, q)$ over finite fields has a long history. For small orders such as $(q = 2, 3, 4, 5, 7, 8, 9)$, complete classifications of points, lines, planes, and line spreads have been established. For larger prime orders $(q = 11, 13, 17)$, researchers have investigated line spreads, blocking sets, and other combinatorial structures, providing insight into the geometric and algebraic properties of these spaces [1, 2, 7, 11]. In the case of $(q = 16)$, non-classical spreads and semi field planes have been discovered, revealing more complex configurations [7, 10, 13, 16]. These previous results form the foundation for constructing and analyzing (k, L) -spreads in $PG(3, q)$, as presented in this work.

A projective 3-space $PG(3, q)$ over a field K is a three – dimensional projective space which consists of points, lines and planes with the incidence relation between them.

The projective 3-space satisfies the following axioms [1, 2, 7, 10, 14, 15]:

- A. Any two distinct points are contained in a unique line.
- B. Any three distinct non-collinear points, or any line and point not on the line, are contained in a unique plane.
- C. Any two distinct coplanar lines intersect in a unique point.

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D. Any line not on a given plane intersects the plane in a unique point.

E. Any two distinct planes intersect in a unique line.

A projective space $PG(3, q)$ over Galois field $GF(q)$, $q = p^m$, for some prime number p and some integer m , is a three– dimensional projective space[1,2,7,16].

any point in $PG(3, q)$ has the form of a quadruple (x_1, x_2, x_3, x_4) , where x_1, x_2, x_3, x_4 are elements in $GF(q)$ with the exception of the quadruple consisting of four zero elements.[10,12]

Two quadruples (x_1, x_2, x_3, x_4) and (y_1, y_2, y_3, y_4) represent the same point if there exists λ in $GF(q) \setminus \{0\}$ such that $(x_1, x_2, x_3, x_4) = \lambda (y_1, y_2, y_3, y_4)$.

Similarly, any plane in $PG(3, q)$ has the form of a quadruple $[x_1, x_2, x_3, x_4]$, where x_1, x_2, x_3, x_4 are elements in $GF(q)$ are not all zero.

Two quadruples $[x_1, x_2, x_3, x_4]$ and $[y_1, y_2, y_3, y_4]$ represent the same plane if there exists λ in

$GF(q) \setminus \{0\}$ such that $[x_1, x_2, x_3, x_4] = \lambda [y_1, y_2, y_3, y_4]$,

Also a point $P(x_1, x_2, x_3, x_4)$ is incident with the plane $\pi[a_1, a_2, a_3, a_4]$ iff

$$a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 = 0.$$

2. Definition and Theorems [1]

Definition (2.1) [6]: A plane π in $PG(3, q)$ is the set of all points $P(x_1, x_2, x_3, x_4)$ satisfying a linear equation $u_1 x_1 + u_2 x_2 + u_3 x_3 + u_4 x_4 = 0$

This plane is denoted by $\pi [u_1, u_2, u_3, u_4]$. It should be noted that if one takes another representation of P , say $(\lambda x_1, \lambda x_2, \lambda x_3, \lambda x_4)$, where $\lambda \in GF(q) \setminus \{0\}$, then since

$u_1 \lambda x_1 + u_2 \lambda x_2 + u_3 \lambda x_3 + u_4 \lambda x_4 = \lambda (u_1 x_1 + u_2 x_2 + u_3 x_3 + u_4 x_4)$, which shows that the definition of a plane is independent of the choice of representatives of its points.

Theorem (2.2) [2]: The points of $PG(3, q)$ have unique forms $(1, 0, 0, 0), (x, 1, 0, 0), (x, y, 1, 0), (x, y, z, 1)$ for all x, y, z in $GF(q)$.

Theorem (2.3) [2]: The planes of $PG(3, q)$ have unique forms $[1, 0, 0, 0], [x, 1, 0, 0], [x, y, 1, 0], [x, y, z, 1]$ for all x, y, z in $GF(q)$.

Theorem (2.4) [4]: Every line in $PG(3, q)$ contains exactly $q + 1$ points.

Theorem (2.5) [9]: Every point in $PG(3, q)$ is on exactly $q + 1$ lines.

Theorem (2.6) [6]: Every plane in $PG(3, q)$ contains exactly $q^2 + q + 1$ points (lines).

Theorem (2.7) [6]: Every point in $PG(3, q)$ is on exactly $q^2 + q + 1$ planes.

Theorem (2.8) [8]: There exist $q^3 + q^2 + q + 1$ points in $PG(3, q)$.

Theorem (2.9) [8]: There exist $q^3 + q^2 + q + 1$ planes in $PG(3, q)$.

Theorem (2.10) [8]: Any two planes in $PG(3, q)$ intersect in exactly $q + 1$ points.

Theorem (2.11) [8]: Any line in $PG(3, q)$ is on exactly $q + 1$ planes.

Theorem (2.12) [8]: Any two points in $PG(3, q)$ are on exactly $q + 1$ planes.

Theorem (2.13) [3]: There exist $(q^2 + 1)(q^2 + q + 1)$ lines in $PG(3, q)$.

Definition (2.14): [5] A (K, L) – spread, $L \geq 1$ is a set of K spaces π_l no two of which intersect; in particular cases, a maximum (K, L) – spread is a spread. Only the lines of $PG(3, q)$ one can take since there are some non-intersecting lines whereas any two planes are intersected.

Definition (2.15) [5] : A maximum $A(k, L)$ – spread, $L \geq 1$ is a set of k spaces π_l which are every points of $PG(3, q)$ lies in exactly one line of π_l , and every two lines of π_l are disjoint.

Definition (2.16) [5]: Every maximum (k, L) – spread is a span.

3. The Projective Space and The (k, L) – spread in $PG(3, 19)$

3.1. The Projective Space $PG(3, 19)$

$PG(3,19)$ contains 7240 points and 7240 planes such that each point lies on 381 planes and every plane contains 381 points ,any line contains 20 points and it is the incident with 20 planes, all the point, planes and lines of $PG(3,19)$ are given in tables (2)

3.2. Method to Calculate Points, Lines, and Planes in $PG(3, 19)$

The projective space $PG(3,19)$ is a three-dimensional space over the finite field $GF(19)$. The points, lines, and planes can be systematically calculated using the combinatorial properties of finite projective spaces as follows:

1. Points:

Each point in $PG(3,19)$ is represented by a quadruple (x_1, x_2, x_3, x_4) where each x_i in $GF(19)$ and not all x_i are zero. Two quadruples (x_1, x_2, x_3, x_4) and (y_1, y_2, y_3, y_4) . Represent the same point if there exists a nonzero λ in $GF(19)$ such that [8]:

$$(x_1, x_2, x_3, x_4) = \lambda (y_1, y_2, y_3, y_4).$$

The total number of points is given by:

$$N_{point} = \frac{(19)^4 - 1}{19 - 1} = 7240$$

2. Lines:

A line in $PG(3,19)$ is determined by any two distinct points. The total number of lines can be calculated using the Gaussian binomial coefficient [3, 8]:

$$N_{lines} = \frac{((19)^4 - 1)((19)^4 - 19)}{((19)^2 - 1)((19)^2 - 19)} = 7240$$

Each line contains 20 points and is incident with 20 planes.

3. Planes:

A plane in $PG(3,19)$ can be represented by a quadruple $[a_1, a_2, a_3, a_4]$, not all zero. Two quadruples represent the same plane if there exists a nonzero λ in $GF(19)$ such that [8]:

$$[x_1, x_2, x_3, x_4] = \lambda [y_1, y_2, y_3, y_4].$$

The total number of planes equals the number of points:

$$N_{planes} = 7240$$

4. Incidence Relations:

- A. Each point lies on 381 planes.
- B. Each plane contains 381 points.
- C. Each line contains 20 points and intersects 20 planes.

5. Procedural Steps for Generating Tables of Points, Lines, and Planes:

1. Generate all nonzero quadruples over $GF(19)$.
2. Reduce quadruples by scalar multiplication to identify unique points and planes.
3. Construct lines by pairing points that do not lie on the same line already.
4. Determine incidences by checking linear dependencies for points on lines and planes.
5. Populate tables systematically to represent all 7240 points, lines, and planes and their incidences.

This method ensures that all points, lines, and planes in $PG(3,19)$ are counted correctly and their incidences are represented completely, as shown in Table 2.

3.3. The (k, L) – spread in the projective geometry $PG(3, 19)$

Example of Numerical Forms in Vector Form in $PG(3,19)$:

In $PG(3,19)$, each point can be represented as a quadruple (x_1, x_2, x_3, x_4) over $GF(19)$, not all zero. Two quadruples represent the same point if they differ by a nonzero scalar multiple in $GF(19)$.

Here are some examples of points in vector form:

- Point 1: $(1 = (1,0,0,0))$
- Point 2: $(2 = (0,1,0,0))$
- Point 3: $(3 = (0,0,1,0))$
- Point 4: $(4 = (0,0,0,1))$
- ...
- Point 20: $(20 = (1,1,1,1))$
- ...
- Point 7240: $(7240 = (18,18,18,18))$

Each point in $PG(3,19)$ can be scaled by any nonzero element λ in $GF(19)$. For instance, $(1,0,0,0)$ represents the same point as $(2,0,0,0)$, $(3,0,0,0)$, ... up to $(18,0,0,0)$.

Method to assign vector forms to points:

1. List all quadruples (x_1, x_2, x_3, x_4) with entries from $GF(19)$, not all zero.
2. Reduce by the scalar multiplication equivalence to obtain unique points.
3. Assign numbers from 1 to 7240 sequentially to these unique points.

This approach allows representing lines and spreads numerically as sets of point vectors, which is used in the tables for $PG(3,19)$.

In table (1), Any two non-intersecting lines can be taken $PG(3,19)$, say and

L_1	1	2	3	4	5	6	7	8	9	10	11	12	13
	14	15	16	17	18	19	20						

then $A_1 = \{ L_1, L_2 \}$ is $(2, L)$ -spread. One can add another line

L_2	21	382	743	1104	1465	1826	2187	2548	2909	3270	3631	3992	4353
	4714	5075	5436	5797	6158	6519	6880						

then $A_2=\{L_1, L_2, L_3\}$ is a $(3, L)$ -span, since L_3 cannot intersect L_1 or L_2 . The line

L_3	22	401	763	1125	1487	1849	2211	2573	2935	3297	3659	4021	4383
	4745	5107	5469	5831	6193	6555	6917						

This line cannot intersect any line of A_2 , then $A_3=A_2 \cup L_4=\{L_1, L_2, L_3, L_4\}$ is a $(4, L)$ -spread. The line

L_4	23	420	783	1146	1509	1872	2235	2598	2961	3324	3687	4031	4394
	4757	5120	5483	5846	6209	6572	6935						

This line cannot intersect any line of A_3 , then $A_4=A_3 \cup L_5=\{L_1, L_2, L_3, L_4, L_5\}$ is a $(5, L)$ -spread. The line

L_5	24	439	803	1167	1531	1895	2259	2623	2968	3332	3696	4060	4424
	4788	5133	5497	5861	6225	6589	6953						

this line cannot intersect any line of A_4 , then $A_5=A_4 \cup L_6=\{L_1, L_2, L_3, L_4, L_5, L_6\}$ is a $(6, L)$ -spread. The line

L_6	25	458	823	1188	1553	1918	2264	2629	2994	3359	3724	4070	4435
	4800	5165	5530	5876	6241	6606	6971						

this line cannot intersect any line of A_4 , then $A_5=A_4 \cup L_6=\{L_1, L_2, L_3, L_4, L_5, L_6, L_7\}$ is a $(7, L)$ -spread. The line

L_7	26	477	843	1209	1575	1922	2288	2654	3020	3367	3733	4099	4465
	4812	5178	5544	5910	6257	6623	6989						

this line cannot intersect any line of A_5 , then $A_6=A_5 \cup L_7=\{L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8\}$

L_8	27	496	836	1230	1597	1945	2312	2679	3027	3394	3761	41019	4476
	4843	5191	5558	5925	6273	6640	7007						

Table 1: for The (k, L) – spread in $PG(3,19)$ is a $(8, L)$ -spread. And we continue into the end lines ,we obtain in table (1)

Table (1) for The (k, L) – spread in $PG(3,19)$

L_1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
L_2	21	382	743	1104	1465	1826	2187	2548	2909	3270	3631	3992	4353	4714	5075	5436	5797	6158	6519	6880
L_3	22	401	763	1125	1487	1849	2211	2573	2935	3297	3659	4021	4383	4745	5107	5469	5831	6193	6555	6917
L_4	23	420	783	1146	1509	1872	2235	2598	2961	3324	3687	4031	4394	4757	5120	5483	5846	6209	6572	6935
L_5	24	439	803	1167	1531	1895	2259	2623	2968	3332	3696	4060	4424	4788	5133	5497	5861	6225	6589	6953
L_6	25	458	823	1188	1553	1918	2264	2629	2994	3359	3724	4070	4435	4800	5165	5530	5876	6241	6606	6971
L_7	26	477	843	1209	1575	1922	2288	2654	3020	3367	3733	4099	4465	4812	5178	5544	5910	6257	6623	6989
L_8	27	496	863	1230	1597	1945	2312	2679	3027	3394	3761	4109	4476	4843	5191	5558	5925	6273	6640	7007
L_9	28	515	883	1251	1600	1968	2336	2685	3053	3421	3770	4138	4487	4855	5223	5572	5940	6308	6657	7025
L_{10}	29	534	903	1272	1622	1991	2341	2710	3079	3429	3798	4148	4517	4867	5236	5605	5955	6324	6674	7043
L_{11}	30	553	923	1293	1644	2014	2365	2735	3086	3456	3807	4177	4528	4898	5249	5619	5970	6340	6691	7061
L_{12}	31	572	943	1295	1666	2018	2389	2741	3112	3464	3835	4187	4558	4910	5281	5633	6004	6356	6727	7079

L13	32	591	963	1316	1688	2041	2413	2766	3119	3491	3844	4216	4569	4941	5294	5647	6019	6372	6744	7097
L14	33	610	983	1337	1710	2064	2418	2791	3145	3499	3872	4226	4599	4953	5307	5680	6034	6388	6761	7115
L15	34	629	1003	1358	1713	2087	2442	2797	3171	3526	3881	4255	4610	4965	5339	5694	6049	6423	6778	7133
L16	35	648	1023	1379	1735	2110	2466	2822	3178	3553	3909	4265	4621	4996	5352	5708	6064	6439	6795	7151
L17	36	667	1043	1400	1757	2114	2490	2847	3204	3561	3918	4294	4651	5008	5365	5722	6098	6455	6812	7169
L18	37	686	1063	1421	1779	2137	2495	2853	3230	3588	3946	4304	4662	5020	5397	5755	6113	6471	6829	7187
L19	38	705	1083	1442	1801	2160	2519	2878	3237	3596	3955	4333	4692	5051	5410	5769	6128	6487	6846	7205
L20	39	724	1103	1463	1823	2183	2543	2903	3263	3623	3983	4343	4703	5063	5423	5783	6143	6503	6863	7223
L21	40	383	744	1105	1466	1827	2188	2549	2910	3271	3632	3993	4354	4715	5076	5437	5798	6159	6520	6881
L22	41	402	762	1124	1485	1846	2207	2568	2929	3290	3651	4012	4373	4734	5095	5456	5817	6178	6539	6900
L23	42	421	782	1143	1504	1865	2226	2587	2948	3309	3670	4030	4392	4753	5114	5475	5836	6197	6558	6919
L24	43	440	801	1162	1523	1884	2245	2606	2967	3328	3689	4050	4411	4772	5132	5494	5855	6216	6577	6938
L25	44	459	820	1181	1542	1903	2263	2625	2986	3347	3708	4069	4430	4791	5152	5513	5874	6235	6596	6957
L26	45	478	839	1200	1561	1921	2283	2644	3005	3366	3727	4088	4449	4810	5171	5532	5893	6254	6615	6976
L27	46	497	858	1219	1580	1941	2302	2663	3024	3385	3746	4107	4468	4829	5190	5551	5912	6272	6634	6995
L28	47	516	877	1238	1599	1960	2321	2682	3043	3404	3765	4126	4486	4848	5209	5570	5931	6292	6653	7014
L29	48	535	896	1257	1618	1979	2340	2701	3062	3423	3784	4145	4506	4866	5228	5589	5950	6311	6672	7033
L30	49	554	915	1276	1637	1998	2359	2720	3081	3442	3803	4164	4525	4886	5247	5608	5969	6330	6690	7052
L31	50	573	934	1294	1656	2017	2378	2739	3100	3461	3822	4183	4544	4905	5266	5627	5988	6349	6710	7071
L32	51	592	953	1314	1675	2036	2397	2758	3118	3480	3841	4202	4563	4924	5285	5646	6007	6368	6729	7090
L33	52	611	972	1333	1694	2055	2416	2777	3138	3498	3860	4221	4582	4943	5304	5665	6026	6387	6748	7109
L34	53	630	991	1352	1712	2074	2435	2796	3157	3518	3879	4240	4601	4962	5323	5684	6045	6406	6767	7128
L35	54	649	1010	1371	1732	2093	2454	2815	3176	3537	3898	4259	4620	4981	5342	5703	6063	6425	6786	7147
L36	55	668	1029	1390	1751	2112	2473	2834	3195	3556	3917	4278	4639	5000	5361	5721	6083	6444	6805	7166
L37	56	687	1048	1409	1770	2131	2492	2852	3214	3575	3936	4297	4658	5019	5380	5741	6102	6463	6824	7185
L38	57	706	1067	1428	1789	2150	2511	2872	3233	3594	3954	4316	4677	5038	5399	5760	6121	6482	6843	7204
L39	58	725	1086	1447	1808	2169	2530	2891	3252	3613	3974	4335	4696	5057	5418	5779	6140	6501	6862	7222
L40	59	384	745	1106	1467	1828	2189	2550	2911	3272	3633	3994	4355	4716	5077	5438	5799	6160	6521	6882
L41	60	403	764	1123	1486	1847	2208	2569	2930	3291	3652	4013	4374	4735	5096	5457	5818	6179	6540	6901
L42	61	422	781	1144	1505	1866	2227	2588	2949	3310	3671	4032	4393	4754	5115	5476	5837	6198	6559	6920
L43	62	441	802	1163	1524	1885	2246	2607	2966	3329	3690	4051	4412	4773	5134	5495	5856	6217	6578	6939
L44	63	460	821	1182	1543	1904	2265	2626	2987	3348	3709	4068	4431	4792	5153	5514	5875	6236	6597	6958
L45	64	479	840	1201	1562	1923	2284	2645	3006	3365	3728	4089	4450	4811	5172	5533	5894	6255	6616	6977
L46	65	498	859	1220	1581	1942	2303	2664	3025	3386	3747	4108	4469	4830	5189	5552	5913	6274	6635	6996
L47	66	517	878	1239	1598	1961	2322	2683	3044	3405	3766	4127	4488	4849	5210	5571	5932	6293	6654	7015
L48	67	536	897	1258	1619	1980	2339	2702	3063	3424	3785	4146	4507	4868	5229	5590	5951	6312	6673	7034
L49	68	555	916	1277	1638	1999	2360	2721	3082	3443	3804	4165	4526	4887	5248	5609	5968	6331	6692	7053
L50	69	574	935	1296	1657	2016	2379	2740	3101	3462	3823	4184	4545	4906	5267	5628	5989	6350	6711	7072
L51	70	593	954	1315	1676	2037	2398	2759	3120	3481	3842	4203	4564	4925	5286	5645	6008	6369	6730	7091
L52	71	612	973	1334	1695	2056	2417	2778	3139	3500	3861	4222	4583	4944	5305	5666	6027	6386	6749	7110
L53	72	631	992	1353	1714	2075	2436	2795	3158	3519	3880	4241	4602	4963	5324	5685	6046	6407	6768	7129
L54	73	650	1011	1372	1733	2094	2455	2816	3177	3538	3899	4260	4619	4982	5343	5704	6065	6426	6787	7148
L55	74	669	1030	1391	1752	2113	2474	2835	3196	3557	3916	4279	4640	5001	5362	5723	6084	6445	6806	7167
L56	75	688	1049	1410	1771	2132	2493	2854	3215	3576	3937	4298	4659	5018	5381	5742	6103	6464	6825	7186
L57	76	707	1068	1429	1790	2151	2512	2873	3234	3595	3956	4317	4678	5039	5400	5761	6122	6483	6844	7203
L58	77	726	1087	1448	1809	2170	2531	2892	3253	3614	3975	4336	4697	5058	5419	5780	6141	6502	6861	7224
L59	78	385	746	1107	1468	1829	2190	2551	2912	3273	3634	3995	4356	4717	5078	5439	5800	6161	6522	6883
L60	79	404	765	1126	1484	1848	2209	2570	2931	3292	3653	4014	4375	4736	5097	5458	5819	6180	6541	6902
L61	80	423	784	1145	1506	1867	2228	2589	2950	3311	3672	4033	4391	4755	5116	5477	5838	6199	6560	6921
L62	81	442	800	1164	1525	1886	2247	2608	2969	3330	3691	4052	4413	4774	5135	5496	5857	6218	6579	6940
L63	82	461	822	1183	1544	1905	2266	2627	2988	3349	3710	4071	4432	4793	5154	5515	5873	6237	6598	6959
L64	83	480	841	1202	1563	1924	2285	2646	3007	3368	3729	4090	4451	4809	5173	5534	5895	6256	6617	6978
L65	84	499	860	1221	1582	1943	2304	2665	3026	3387	3748	4106	4470	4831	5192	5553	5914	6275	6636	6997
L66	85	518	879	1240	1601	1962	2323	2684	3045	3406	3767	4128	4489	4850	5211	5569	5933	6294	6655	7016
L67	86	537	898	1259	1620	1981	2342	2703	3064	3425	3786	4147	4508	4869	5230	5591	5952	6313	6671	7035

L68	87	556	917	1278	1639	2000	2361	2722	3083	3444	3805	4166	4527	4888	5246	5610	5971	6332	6693	7054
L69	88	575	936	1297	1658	2019	2380	2738	3102	3463	3824	4185	4546	4907	5268	5629	5990	6351	6712	7073
L70	89	594	955	1313	1677	2038	2399	2760	3121	3482	3843	4204	4565	4926	5287	5648	6009	6370	6731	7092
L71	90	613	974	1335	1696	2057	2415	2779	3140	3501	3862	4223	4584	4945	5306	5667	6028	6389	6750	7111
L72	91	632	993	1354	1715	2076	2437	2798	3159	3520	3878	4242	4603	4964	5325	5686	6047	6408	6769	7130
L73	92	651	1012	1373	1734	2095	2456	2817	3175	3539	3900	4261	4622	4983	5344	5705	6066	6427	6788	7149
L74	93	670	1031	1392	1753	2111	2475	2836	3197	3558	3919	4280	4641	5002	5363	5724	6085	6446	6807	7168
L75	94	689	1050	1411	1772	2133	2494	2855	3216	3577	3938	4299	4660	5021	5382	5743	6104	6465	6826	7184
L76	95	708	1069	1430	1791	2152	2513	2874	3235	3593	3957	4318	4679	5040	5401	5762	6123	6484	6845	7206
L77	96	727	1088	1449	1810	2171	2532	2893	3254	3615	3976	4337	4698	5059	5420	5781	6142	6500	6864	7225
L78	97	386	747	1108	1469	1830	2191	2552	2913	3274	3635	3996	4357	4718	5079	5440	5801	6162	6523	6884
L79	98	405	766	1127	1488	1845	2210	2571	2932	3293	3654	4015	4376	4737	5098	5459	5820	6181	6542	6903
L80	99	424	785	1142	1507	1868	2229	2590	2951	3312	3673	4034	4395	4756	5117	5478	5839	6200	6561	6922
L81	100	443	804	1165	1526	1887	2248	2609	2970	3331	3692	4053	4414	4775	5136	5493	5858	6219	6580	6941
L82	101	462	819	1184	1545	1906	2267	2628	2989	3350	3711	4072	4433	4794	5155	5516	5877	6238	6599	6960
L83	102	481	842	1203	1564	1925	2286	2647	3008	3369	3730	4091	4452	4813	5174	5535	5896	6253	6618	6979
L84	103	500	861	1222	1583	1944	2305	2666	3023	3388	3749	4110	4471	4832	5193	5554	5915	6276	6637	6998
L85	104	519	880	1241	1602	1963	2324	2681	3046	3407	3768	4129	4490	4851	5212	5573	5934	6295	6656	7017
L86	105	538	899	1260	1621	1982	2343	2704	3065	3426	3787	4144	4509	4870	5231	5592	5953	6314	6675	7036
L87	106	557	918	1279	1640	2001	2362	2723	3084	3445	3806	4167	4524	4889	5250	5611	5972	6333	6694	7055
L88	107	576	937	1298	1659	2020	2381	2742	3103	3460	3825	4186	4547	4908	5269	5630	5991	6352	6713	7074
L89	108	595	956	1317	1678	2039	2400	2761	3122	3483	3840	4205	4566	4927	5288	5649	6010	6371	6732	7093
L90	109	614	975	1336	1697	2058	2419	2780	3141	3502	3863	4224	4585	4946	5303	5668	6029	6390	6751	7112
L91	110	633	994	1355	1716	2077	2438	2799	3160	3521	3882	4243	4604	4961	5326	5687	6048	6409	6770	7131
L92	111	652	1013	1374	1731	2096	2457	2818	3179	3540	3901	4262	4623	4984	5345	5706	6067	6428	6789	7150
L93	112	671	1032	1393	1754	2115	2476	2837	3198	3559	3920	4281	4642	5003	5364	5725	6086	6447	6808	7165
L94	113	690	1051	1412	1773	2134	2491	2856	3217	3578	3939	4300	4661	5022	5383	5744	6105	6466	6827	7188
L95	114	709	1070	1431	1792	2153	2514	2875	3236	3597	3958	4319	4680	5041	5402	5763	6124	6485	6842	7207
L96	115	728	1089	1450	1811	2172	2533	2894	3255	3616	3977	4338	4699	5060	5421	5782	6139	6504	6865	7226
L97	116	387	748	1109	1470	1831	2192	2553	2914	3275	3636	3997	4358	4719	5080	5441	5802	6163	6524	6885
L98	117	406	767	1128	1489	1850	2206	2572	2933	3294	3655	4016	4377	4738	5099	5460	5821	6182	6543	6904
L99	118	425	786	1147	1508	1869	2230	2591	2952	3313	3674	4035	4396	4752	5118	5479	5840	6201	6562	6923
L100	119	444	805	1166	1527	1888	2249	2610	2971	3327	3693	4054	4415	4776	5137	5498	5859	6220	6581	6942
L101	120	463	824	1185	1546	1907	2268	2624	2990	3351	3712	4073	4434	4795	5156	5517	5878	6239	6600	6961
L102	121	482	838	1204	1565	1926	2287	2648	3009	3370	3731	4092	4453	4814	5175	5536	5897	6258	6619	6980
L103	122	501	862	1223	1584	1940	2306	2667	3028	3389	3750	4111	4472	4833	5194	5555	5916	6277	6638	6999
L104	123	520	881	1242	1603	1964	2325	2686	3047	3408	3769	4130	4491	4852	5213	5574	5935	6296	6652	7018
L105	124	539	900	1261	1617	1983	2344	2705	3066	3427	3788	4149	4510	4871	5232	5593	5954	6315	6676	7037
L106	125	558	919	1280	1641	2002	2363	2724	3085	3446	3802	4168	4529	4890	5251	5612	5973	6334	6695	7056
L107	126	577	938	1299	1660	2021	2382	2743	3104	3465	3826	4182	4548	4909	5270	5631	5992	6353	6714	7075
L108	127	596	957	1318	1679	2040	2401	2762	3123	3484	3845	4206	4567	4928	5289	5650	6011	6367	6733	7094
L109	128	615	976	1332	1698	2059	2420	2781	3142	3503	3864	4225	4586	4947	5308	5669	6030	6391	6752	7113
L110	129	634	995	1356	1717	2078	2439	2800	3161	3522	3883	4244	4605	4966	5327	5688	6044	6410	6771	7132
L111	130	653	1014	1375	1736	2097	2458	2819	3180	3541	3902	4263	4624	4985	5346	5707	6068	6429	6790	7146
L112	131	672	1033	1394	1755	2116	2477	2838	3199	3560	3921	4282	4643	5004	5360	5726	6087	6448	6809	7170
L113	132	691	1052	1413	1774	2135	2496	2857	3218	3579	3940	4301	4657	5023	5384	5745	6106	6467	6828	7189
L114	133	710	1071	1432	1793	2154	2515	2876	3232	3598	3959	4320	4681	5042	5403	5764	6125	6486	6847	7208
L115	134	729	1090	1451	1812	2173	2534	2895	3256	3617	3978	4339	4700	5061	5422	5778	6144	6505	6866	7227
L116	135	388	749	1110	1471	1832	2193	2554	2915	3276	3637	3998	4359	4720	5081	5442	5803	6164	6525	6886
L117	136	407	768	1129	1490	1851	2212	2567	2934	3295	3656	4017	4378	4739	5100	5461	5822	6183	6544	6905
L118	137	426	787	1148	1503	1870	2231	2592	2953	3314	3675	4036	4397	4758	5119	5480	5841	6202	6563	6924
L119	138	445	806	1161	1528	1889	2250	2611	2972	3333	3694	4055	4416	4777	5138	5499	5860	6221	6582	6943
L120	139	464	825	1186	1547	1908	2269	2630	2991	3352	3713	4074	4429	4796	5157	5518	5879	6240	6601	6962
L121	140	483	844	1205	1566	1927	2282	2649	3010	3371	3732	4093	4454	4815	5176	5537	5898	6259	6620	6981
L122	141	502	857	1224	1585	1946	2307	2668	3029	3390	3751	4112	4473	4834	5195	5556	5917	6278	6639	7000

L123	142	521	882	1243	1604	1965	2326	2687	3048	3409	3764	4131	4492	4853	5214	5575	5936	6297	6658	7019
L124	143	540	901	1262	1623	1984	2345	2706	3067	3428	3789	4150	4511	4872	5233	5594	5949	6316	6677	7038
L125	144	559	920	1281	1642	2003	2364	2725	3080	3447	3808	4169	4530	4891	5252	5613	5974	6335	6696	7057
L126	145	578	939	1300	1661	2022	2383	2744	3105	3466	3827	4188	4549	4904	5271	5632	5993	6354	6715	7076
L127	146	597	958	1319	1680	2035	2402	2763	3124	3485	3846	4207	4568	4929	5290	5651	6012	6373	6734	7095
L128	147	616	977	1338	1699	2060	2421	2782	3143	3504	3865	4220	4587	4948	5309	5670	6031	6392	6753	7114
L129	148	635	996	1357	1718	2079	2440	2801	3162	3523	3884	4245	4606	4967	5328	5689	6050	6411	6772	7127
L130	149	654	1015	1376	1737	2098	2459	2820	3181	3542	3903	4264	4625	4986	5347	5702	6069	6430	6791	7152
L131	150	673	1034	1395	1756	2117	2478	2839	3200	3555	3922	4283	4644	5005	5366	5727	6088	6449	6810	7171
L132	151	692	1053	1414	1775	2136	2497	2858	3219	3580	3941	4302	4663	5024	5385	5746	6107	6468	6823	7190
L133	152	711	1072	1433	1794	2155	2516	2877	3238	3599	3960	4321	4682	5043	5404	5765	6126	6481	6848	7209
L134	153	730	1091	1452	1813	2174	2535	2896	3257	3618	3979	4340	4701	5062	5417	5784	6145	6506	6867	7228
L135	154	389	750	1111	1472	1833	2194	2555	2916	3277	3638	3999	4360	4721	5082	5443	5804	6165	6526	6887
L136	155	408	769	1130	1491	1852	2213	2574	2928	3296	3657	4018	4379	4740	5101	5462	5823	6184	6545	6906
L137	156	427	788	1149	1510	1871	2232	2593	2954	3315	3676	4037	4398	4759	5113	5481	5842	6203	6564	6925
L138	157	446	807	1168	1529	1890	2251	2612	2973	3334	3695	4056	4417	4778	5139	5500	5854	6222	6583	6944
L139	158	465	826	1187	1548	1909	2270	2631	2992	3353	3714	4075	4436	4797	5158	5519	5880	6234	6602	6963
L140	159	484	845	1206	1567	1928	2289	2650	3011	3372	3726	4094	4455	4816	5177	5538	5899	6260	6621	6982
L141	160	503	864	1225	1586	1947	2308	2669	3030	3391	3752	4113	4474	4835	5196	5557	5918	6279	6633	7001
L142	161	522	876	1244	1605	1966	2327	2688	3049	3410	3771	4132	4493	4854	5215	5576	5937	6298	6659	7020
L143	162	541	902	1263	1624	1985	2346	2707	3068	3422	3790	4151	4512	4873	5234	5595	5956	6317	6678	7039
L144	163	560	921	1282	1643	2004	2358	2726	3087	3448	3809	4170	4531	4892	5253	5614	5975	6336	6697	7058
L145	164	579	940	1301	1662	2023	2384	2745	3106	3467	3828	4189	4550	4911	5272	5626	5994	6355	6716	7077
L146	165	598	959	1320	1681	2042	2403	2764	3125	3486	3847	4208	4562	4930	5291	5652	6013	6374	6735	7096
L147	166	617	978	1339	1700	2061	2422	2783	3144	3505	3866	4227	4588	4949	5310	5671	6032	6393	6754	7108
L148	167	636	997	1351	1719	2080	2441	2802	3163	3524	3885	4246	4607	4968	5329	5690	6051	6412	6773	7134
L149	168	655	1016	1377	1738	2099	2460	2821	3182	3543	3904	4258	4626	4987	5348	5709	6070	6431	6792	7153
L150	169	674	1035	1396	1750	2118	2479	2840	3201	3562	3923	4284	4645	5006	5367	5728	6089	6450	6811	7172
L151	170	693	1054	1415	1776	2130	2498	2859	3220	3581	3942	4303	4664	5025	5386	5747	6108	6469	6830	7191
L152	171	712	1073	1434	1795	2156	2517	2871	3239	3600	3961	4322	4683	5044	5405	5766	6127	6488	6849	7210
L153	172	731	1092	1453	1814	2175	2536	2897	3258	3619	3980	4341	4702	5056	5424	5785	6146	6507	6868	7229
L154	173	390	751	1112	1473	1834	2195	2556	2917	3278	3639	4000	4361	4722	5083	5444	5805	6166	6527	6888
L155	174	409	770	1131	1492	1853	2214	2575	2936	3289	3658	4019	4380	4741	5102	5463	5824	6185	6546	6907
L156	175	428	789	1150	1511	1864	2233	2594	2955	3316	3677	4038	4399	4760	5121	5482	5843	6204	6565	6926
L157	176	447	808	1169	1530	1891	2252	2613	2974	3335	3688	4057	4418	4779	5140	5501	5862	6223	6584	6945
L158	177	466	827	1180	1549	1910	2271	2632	2993	3354	3715	4076	4437	4798	5159	5520	5881	6242	6603	6964
L159	178	485	846	1207	1568	1929	2290	2651	3012	3373	3734	4095	4456	4817	5170	5539	5900	6261	6622	6983
L160	179	504	865	1226	1587	1948	2309	2670	3031	3392	3753	4114	4475	4836	5197	5550	5919	6280	6641	7002
L161	180	523	884	1245	1606	1967	2328	2689	3050	3411	3772	4133	4494	4847	5216	5577	5938	6299	6660	7021
L162	181	542	895	1264	1625	1986	2347	2708	3069	3430	3791	4152	4513	4874	5235	5596	5957	6318	6679	7040
L163	182	561	922	1283	1636	2005	2366	2727	3088	3449	3810	4171	4532	4893	5254	5615	5976	6337	6698	7059
L164	183	580	941	1302	1663	2024	2385	2746	3107	3468	3829	4190	4551	4912	5273	5634	5995	6348	6717	7078
L165	184	599	960	1321	1682	2043	2404	2765	3126	3487	3848	4209	4570	4931	5292	5653	6014	6375	6736	7089
L166	185	618	979	1340	1701	2062	2423	2784	3137	3506	3867	4228	4589	4950	5311	5672	6033	6394	6755	7116
L167	186	637	998	1359	1720	2081	2434	2803	3164	3525	3886	4247	4608	4969	5330	5691	6052	6413	6774	7135
L168	187	656	1017	1378	1739	2100	2461	2814	3183	3544	3905	4266	4627	4988	5349	5710	6071	6432	6793	7154
L169	188	675	1036	1397	1758	2119	2480	2841	3202	3563	3924	4285	4646	5007	5368	5729	6090	6451	6804	7173
L170	189	694	1055	1416	1777	2138	2499	2860	3221	3582	3943	4296	4665	5026	5387	5748	6109	6470	6831	7192
L171	190	713	1074	1435	1796	2157	2518	2879	3240	3601	3962	4323	4684	5045	5406	5767	6120	6489	6850	7211
L172	191	732	1093	1454	1815	2176	2537	2898	3259	3620	3981	4342	4695	5064	5425	5786	6147	6508	6869	7230
L173	192	391	752	1113	1474	1835	2196	2557	2918	3279	3640	4001	4362	4723	5084	5445	5806	6167	6528	6889
L174	193	410	771	1132	1493	1854	2215	2576	2937	3298	3650	4020	4381	4742	5103	5464	5825	6186	6547	6908
L175	194	429	790	1151	1512	1873	2234	2595	2956	3317	3678	4039	4400	4761	5122	5474	5844	6205	6566	6927
L176	195	448	809	1170	1521	1892	2253	2614	2975	3336	3697	4058	4419	4780	5141	5502	5863	6224	6585	6946
L177	196	467	828	1189	1550	1911	2272	2633	2985	3355	3716	4077	4438	4799	5160	5521	5882	6243	6604	6965

L178	197	486	847	1208	1569	1930	2291	2652	3013	3374	3735	4096	4457	4818	5179	5540	5901	6262	6614	6984
L179	198	505	866	1227	1588	1949	2310	2671	3032	3393	3754	4115	4467	4837	5198	5559	5920	6281	6642	7003
L180	199	524	885	1246	1607	1959	2329	2690	3051	3412	3773	4134	4495	4856	5217	5578	5939	6300	6661	7022
L181	200	543	904	1265	1626	1987	2348	2709	3070	3431	3792	4153	4514	4875	5227	5597	5958	6319	6680	7041
L182	201	562	914	1284	1645	2006	2367	2728	3089	3450	3811	4172	4533	4894	5255	5616	5977	6338	6699	7060
L183	202	581	942	1303	1664	2025	2386	2747	3108	3469	3830	4191	4552	4913	5274	5635	5996	6357	6718	7079
L184	203	600	961	1322	1683	2044	2405	2757	3127	3488	3849	4210	4571	4932	5293	5654	6015	6376	6737	7098
L185	204	619	980	1341	1702	2063	2424	2785	3146	3507	3868	4229	4590	4951	5312	5673	6025	6395	6756	7117
L186	205	638	999	1360	1721	2082	2443	2804	3165	3517	3887	4248	4609	4970	5331	5692	6053	6414	6775	7136
L187	206	657	1018	1370	1740	2101	2462	2823	3184	3545	3906	4267	4628	4989	5350	5711	6072	6433	6794	7155
L188	207	676	1037	1398	1759	2120	2481	2842	3203	3564	3925	4286	4647	4999	5369	5730	6091	6452	6813	7174
L189	208	695	1056	1417	1778	2139	2500	2861	3222	3583	3944	4305	4666	5027	5388	5749	6110	6462	6832	7193
L190	209	714	1075	1436	1797	2158	2510	2880	3241	3602	3963	4324	4685	5046	5407	5768	6129	6490	6851	7212
L191	210	733	1094	1455	1816	2177	2538	2899	3260	3621	3982	4334	4704	5065	5426	5787	6148	6509	6870	7231
L192	211	392	753	1114	1475	1836	2197	2558	2919	3280	3641	4002	4363	4724	5085	5446	5807	6168	6529	6890
L193	212	411	772	1133	1494	1855	2216	2577	2938	3299	3660	4011	4382	4743	5104	5465	5826	6187	6548	6909
L194	213	430	791	1152	1513	1874	2225	2596	2957	3318	3679	4040	4401	4762	5123	5484	5845	6206	6567	6928
L195	214	449	810	1171	1532	1893	2254	2615	2976	3337	3698	4059	4420	4781	5142	5503	5864	6215	6586	6947
L196	215	468	829	1190	1551	1912	2273	2634	2995	3356	3717	4078	4439	4790	5161	5522	5883	6244	6605	6966
L197	216	487	848	1199	1570	1931	2292	2653	3014	3375	3736	4097	4458	4819	5180	5541	5902	6263	6624	6985
L198	217	506	867	1228	1589	1950	2311	2672	3033	3384	3755	4116	4477	4838	5199	5560	5921	6282	6643	7004
L199	218	525	886	1247	1608	1969	2330	2691	3052	3413	3774	4135	4496	4857	5218	5579	5930	6301	6662	7023
L200	219	544	905	1266	1627	1988	2349	2700	3071	3432	3793	4154	4515	4876	5237	5598	5959	6320	6681	7042
L201	220	563	924	1285	1646	2007	2368	2729	3090	3451	3812	4173	4534	4895	5256	5617	5978	6339	6700	7051
L202	221	582	933	1304	1665	2026	2387	2748	3109	3470	3831	4192	4553	4914	5275	5636	5997	6358	6719	7080
L203	222	601	962	1323	1684	2045	2406	2767	3128	3489	3850	4211	4572	4933	5284	5655	6016	6377	6738	7099
L204	223	620	981	1342	1703	2054	2425	2786	3147	3508	3869	4230	4591	4952	5313	5674	6035	6396	6757	7118
L205	224	639	1000	1361	1722	2083	2444	2805	3166	3527	3888	4249	4600	4971	5332	5693	6054	6415	6776	7137
L206	225	658	1019	1380	1741	2102	2463	2824	3185	3546	3907	4268	4629	4990	5351	5712	6073	6434	6785	7156
L207	226	677	1038	1399	1760	2121	2482	2843	3194	3565	3926	4287	4648	5009	5370	5731	6092	6453	6814	7175
L208	227	696	1057	1418	1769	2140	2501	2862	3223	3584	3945	4306	4667	5028	5389	5750	6111	6472	6833	7194
L209	228	715	1076	1437	1798	2159	2520	2881	3242	3603	3964	4325	4686	5047	5408	5759	6130	6491	6852	7213
L210	229	734	1095	1456	1817	2178	2539	2900	3261	3622	3973	4344	4705	5066	5427	5788	6149	6510	6871	7232
L211	230	393	754	1115	1476	1837	2198	2559	2920	3281	3642	4003	4364	4725	5086	5447	5808	6169	6530	6891
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L213	232	431	792	1153	1514	1875	2236	2597	2958	3319	3680	4041	4402	4763	5124	5485	5835	6207	6568	6929
L214	233	450	811	1172	1533	1894	2255	2616	2977	3338	3699	4049	4421	4782	5143	5504	5865	6226	6587	6948
L215	234	469	830	1191	1552	1913	2274	2635	2996	3357	3718	4079	4440	4801	5162	5523	5884	6245	6595	6967
L216	235	488	849	1210	1571	1932	2293	2643	3015	3376	3737	4098	4459	4820	5181	5542	5903	6264	6625	6986
L217	236	507	868	1229	1590	1951	2301	2673	3034	3395	3756	4117	4478	4839	5200	5561	5922	6283	6644	7005
L218	237	526	887	1248	1609	1970	2331	2692	3042	3414	3775	4136	4497	4858	5219	5580	5941	6302	6663	7024
L219	238	545	906	1267	1628	1989	2350	2711	3072	3433	3794	4155	4516	4877	5238	5599	5960	6321	6682	7032
L220	239	564	925	1286	1647	2008	2369	2730	3091	3452	3813	4174	4535	4896	5257	5618	5979	6329	6701	7062
L221	240	583	944	1305	1655	2027	2388	2749	3110	3471	3832	4193	4554	4915	5276	5637	5998	6359	6720	7081
L222	241	602	952	1324	1685	2046	2407	2768	3129	3490	3851	4212	4573	4934	5295	5656	6017	6378	6739	7100
L223	242	621	982	1343	1704	2065	2426	2787	3148	3509	3870	4231	4592	4942	5314	5675	6036	6397	6758	7119
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L225	244	659	1020	1381	1742	2103	2464	2825	3186	3547	3908	4269	4630	4991	5341	5713	6074	6435	6796	7157
L226	245	678	1039	1389	1761	2122	2483	2844	3205	3566	3927	4288	4649	5010	5371	5732	6093	6454	6815	7176
L227	246	697	1058	1419	1780	2141	2502	2863	3224	3585	3935	4307	4668	5029	5390	5751	6112	6473	6834	7195
L228	247	716	1077	1438	1799	2149	2521	2882	3243	3604	3965	4326	4687	5048	5409	5770	6131	6492	6853	7214
L229	248	735	1096	1457	1818	2179	2540	2901	3262	3612	3984	4345	4706	5067	5428	5789	6150	6511	6872	7233
L230	249	394	755	1116	1477	1838	2199	2560	2921	3282	3643	4004	4365	4726	5087	5448	5809	6170	6531	6892
L231	250	413	774	1135	1496	1857	2218	2579	2940	3301	3662	4023	4384	4733	5106	5467	5828	6189	6550	6911
L232	251	432	793	1154	1515	1876	2237	2586	2959	3320	3681	4042	4403	4764	5125	5486	5847	6208	6569	6930

L233	252	451	812	1173	1534	1883	2256	2617	2978	3339	3700	4061	4422	4783	5144	5505	5866	6227	6588	6949
L234	253	470	831	1192	1541	1914	2275	2636	2997	3358	3719	4080	4441	4802	5163	5524	5885	6246	6607	6968
L235	254	489	850	1211	1572	1933	2294	2655	3016	3377	3738	4087	4460	4821	5182	5543	5904	6265	6626	6987
L236	255	508	869	1218	1591	1952	2313	2674	3035	3396	3757	4118	4479	4840	5201	5562	5923	6284	6645	7006
L237	256	527	888	1249	1610	1971	2332	2693	3054	3415	3776	4137	4498	4859	5220	5581	5942	6303	6664	7013
L238	257	546	907	1268	1629	1990	2351	2712	3073	3434	3795	4156	4505	4878	5239	5600	5961	6322	6683	7044
L239	258	565	926	1287	1648	2009	2370	2731	3092	3453	3814	4175	4536	4897	5258	5607	5980	6341	6702	7063
L240	259	584	945	1306	1667	2028	2377	2750	3111	3472	3833	4194	4555	4916	5277	5638	5999	6360	6721	7082
L241	260	603	964	1325	1686	2047	2408	2769	3130	3479	3852	4213	4574	4935	5296	5657	6018	6379	6740	7101
L242	261	622	971	1344	1705	2066	2427	2788	3149	3510	3871	4232	4593	4954	5315	5676	6037	6398	6759	7120
L243	262	641	1002	1363	1724	2085	2446	2807	3168	3529	3890	4251	4612	4973	5334	5695	6056	6417	6766	7139
L244	263	660	1021	1382	1743	2104	2465	2826	3187	3548	3897	4270	4631	4992	5353	5714	6075	6436	6797	7158
L245	264	679	1040	1401	1762	2123	2484	2845	3206	3567	3928	4289	4650	5011	5372	5733	6094	6443	6816	7177
L246	265	698	1059	1420	1781	2142	2503	2864	3225	3586	3947	4308	4669	5030	5391	5752	6101	6474	6835	7196
L247	266	717	1078	1439	1800	2161	2522	2883	3244	3605	3966	4327	4688	5049	5398	5771	6132	6493	6854	7215
L248	267	736	1097	1458	1819	2180	2541	2902	3251	3624	3985	4346	4707	5068	5429	5790	6151	6512	6873	7234
L249	268	395	756	1117	1478	1839	2200	2561	2922	3283	3644	4005	4366	4727	5088	5449	5810	6171	6532	6893
L250	269	414	775	1136	1497	1858	2219	2580	2941	3302	3663	4024	4385	4746	5094	5468	5829	6190	6551	6912
L251	270	433	794	1155	1516	1877	2238	2599	2960	3321	3682	4043	4404	4765	5126	5487	5848	6196	6570	6931
L252	271	452	813	1174	1535	1896	2257	2618	2979	3340	3701	4062	4423	4784	5145	5506	5867	6228	6576	6950
L253	272	471	832	1193	1554	1915	2276	2637	2998	3346	3720	4081	4442	4803	5164	5525	5886	6247	6608	6969
L254	273	490	851	1212	1573	1934	2295	2656	3017	3378	3739	4100	4461	4822	5183	5531	5905	6266	6627	6988
L255	274	509	870	1231	1592	1953	2314	2675	3036	3397	3758	4119	4480	4841	5202	5563	5924	6285	6646	6994
L256	275	528	889	1250	1611	1972	2333	2694	3055	3416	3777	4125	4499	4860	5221	5582	5943	6304	6665	7026
L257	276	547	908	1269	1630	1978	2352	2713	3074	3435	3796	4157	4518	4879	5240	5601	5962	6323	6684	7045
L258	277	566	927	1288	1649	2010	2371	2732	3093	3454	3815	4176	4537	4885	5259	5620	5981	6342	6703	7064
L259	278	585	946	1307	1668	2029	2390	2751	3099	3473	3834	4195	4556	4917	5278	5639	6000	6361	6722	7083
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L264	283	680	1041	1402	1763	2124	2485	2846	3207	3568	3929	4290	4638	5012	5373	5734	6095	6456	6817	7178
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L266	285	718	1079	1440	1788	2162	2523	2884	3245	3606	3967	4328	4689	5050	5411	5772	6133	6494	6855	7216
L267	286	737	1098	1459	1820	2181	2542	2890	3264	3625	3986	4347	4708	5069	5430	5791	6152	6513	6874	7235
L268	287	396	757	1118	1479	1840	2201	2562	2923	3284	3645	4006	4367	4728	5089	5450	5811	6172	6533	6894
L269	288	415	776	1137	1498	1859	2220	2581	2942	3303	3664	4025	4386	4747	5108	5455	5830	6191	6552	6913
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L271	290	453	814	1175	1536	1897	2258	2619	2980	3341	3702	4063	4410	4785	5146	5507	5868	6229	6590	6951
L272	291	472	833	1194	1555	1916	2277	2638	2999	3360	3721	4082	4443	4804	5151	5526	5887	6248	6609	6970
L273	292	491	852	1213	1574	1935	2296	2657	3018	3379	3740	4101	4462	4823	5184	5545	5906	6267	6628	6975
L274	293	510	871	1232	1593	1954	2315	2676	3037	3398	3759	4120	4481	4842	5203	5564	5911	6286	6647	7008
L275	294	529	890	1237	1612	1973	2334	2695	3056	3417	3778	4139	4500	4861	5222	5583	5944	6305	6666	7027
L276	295	548	909	1270	1631	1992	2353	2714	3075	3436	3797	4158	4519	4880	5241	5602	5963	6310	6685	7046
L277	296	567	928	1289	1650	2011	2372	2733	3094	3455	3816	4163	4538	4899	5260	5621	5982	6343	6704	7065
L278	297	586	947	1308	1669	2030	2391	2752	3113	3474	3821	4196	4557	4918	5279	5640	6001	6362	6723	7084
L279	298	605	966	1327	1674	2049	2410	2771	3132	3493	3854	4215	4576	4937	5298	5659	6020	6381	6742	7103
L280	299	624	985	1346	1707	2068	2429	2790	3151	3512	3873	4234	4595	4956	5317	5678	6039	6400	6747	7122
L281	300	643	1004	1365	1726	2073	2448	2809	3170	3531	3892	4253	4614	4975	5336	5697	6058	6419	6780	7141
L282	301	662	1009	1384	1745	2106	2467	2828	3189	3550	3911	4272	4633	4994	5355	5716	6077	6438	6799	7160
L283	302	681	1042	1403	1764	2125	2486	2833	3208	3569	3930	4291	4652	5013	5374	5735	6096	6457	6818	7179
L284	303	700	1061	1422	1783	2144	2505	2866	3227	3574	3949	4310	4671	5032	5393	5754	6115	6476	6837	7198
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L286	305	738	1099	1460	1821	2182	2529	2904	3265	3626	3987	4348	4709	5070	5431	5792	6153	6514	6875	7236
L287	306	397	758	1119	1480	1841	2202	2563	2924	3285	3646	4007	4368	4729	5090	5451	5812	6173	6534	6895

L288	307	416	777	1138	1499	1860	2221	2582	2943	3304	3665	4026	4387	4748	5109	5470	5816	6192	6553	6914
L289	308	435	796	1157	1518	1879	2240	2601	2962	3323	3684	4045	4406	4767	5128	5489	5850	6211	6557	6933
L290	309	454	815	1176	1537	1898	2244	2620	2981	3342	3703	4064	4425	4786	5147	5508	5869	6230	6591	6952
L291	310	473	834	1195	1556	1917	2278	2639	3000	3361	3722	4083	4444	4805	5166	5527	5888	6249	6610	6956
L292	311	492	853	1214	1560	1936	2297	2658	3019	3380	3741	4102	4463	4824	5185	5546	5907	6268	6629	6990
L293	312	511	872	1233	1594	1955	2316	2677	3038	3399	3760	4121	4482	4828	5204	5565	5926	6287	6648	7009
L294	313	530	891	1252	1613	1974	2335	2696	3057	3418	3779	4140	4501	4862	5208	5584	5945	6306	6667	7028
L295	314	549	910	1271	1632	1993	2354	2715	3076	3437	3783	4159	4520	4881	5242	5603	5964	6325	6686	7047
L296	315	568	929	1290	1651	2012	2373	2734	3095	3441	3817	4178	4539	4900	5261	5622	5983	6344	6705	7066
L297	316	587	948	1309	1670	2031	2392	2753	3114	3475	3836	4197	4543	4919	5280	5641	6002	6363	6724	7085
L298	317	606	967	1328	1689	2050	2411	2772	3133	3494	3855	4201	4577	4938	5299	5660	6021	6382	6743	7104
L299	318	625	986	1347	1708	2069	2430	2776	3152	3513	3874	4235	4596	4957	5318	5679	6040	6401	6762	7123
L300	319	644	1005	1366	1727	2088	2449	2810	3156	3532	3893	4254	4615	4976	5337	5698	6059	6420	6781	7142
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L302	321	682	1028	1404	1765	2126	2487	2848	3209	3570	3931	4292	4653	5014	5375	5736	6097	6458	6819	7180
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L304	323	720	1081	1427	1803	2164	2525	2886	3247	3608	3969	4330	4691	5052	5413	5774	6135	6496	6857	7218
L305	324	739	1100	1461	1822	2168	2544	2905	3266	3627	3988	4349	4710	5071	5432	5793	6154	6515	6876	7237
L306	325	759	1120	1481	1842	2203	2564	2925	3286	3647	4008	4369	4730	5091	5452	5813	6174	6535	6896	
L307	326	417	778	1139	1500	1861	2222	2583	2944	3305	3666	4027	4388	4749	5110	5471	5832	6177	6554	6915
L308	327	436	797	1158	1519	1880	2241	2602	2963	3308	3685	4046	4407	4768	5129	5490	5851	6212	6573	6934
L309	328	455	816	1177	1538	1899	2260	2621	2982	3343	3704	4065	4426	4787	5148	5509	5870	6231	6592	6937
L310	329	474	835	1196	1557	1902	2279	2640	3001	3362	3723	4084	4445	4806	5167	5528	5889	6250	6611	6972
L311	330	493	854	1215	1576	1937	2298	2659	3004	3381	3742	4103	4464	4825	5186	5547	5908	6269	6630	6991
L312	331	512	873	1234	1595	1956	2317	2678	3039	3400	3745	4122	4483	4844	5205	5566	5927	6288	6649	7010
L313	332	531	892	1253	1614	1975	2320	2697	3058	3419	3780	4141	4502	4863	5224	5585	5946	6307	6668	7029
L314	333	550	911	1256	1633	1994	2355	2716	3077	3438	3799	4160	4521	4882	5243	5604	5965	6326	6687	7048
L315	334	569	930	1291	1652	2013	2374	2719	3096	3457	3818	4179	4540	4901	5262	5623	5984	6345	6706	7067
L316	335	588	949	1310	1671	2032	2393	2754	3115	3476	3837	4198	4559	4920	5265	5642	6003	6364	6725	7086
L317	336	607	968	1329	1690	2051	2412	2773	3134	3495	3856	4217	4578	4939	5300	5661	6022	6383	6728	7105
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L319	338	645	1006	1367	1728	2089	2450	2811	3172	3533	3894	4239	4616	4977	5338	5699	6060	6421	6782	7143
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L322	341	702	1047	1424	1785	2146	2507	2868	3229	3590	3951	4312	4673	5034	5395	5756	6117	6478	6839	7200
L323	342	721	1082	1443	1804	2165	2526	2887	3248	3609	3970	4331	4676	5053	5414	5775	6136	6497	6858	7219
L324	343	740	1101	1462	1807	2184	2545	2906	3267	3628	3989	4350	4711	5072	5433	5794	6155	6516	6877	7238
L325	344	399	760	1121	1482	1843	2204	2565	2926	3287	3648	4009	4370	4731	5092	5453	5814	6175	6536	6897
L326	345	418	779	1140	1501	1862	2223	2584	2945	3306	3667	4028	4389	4750	5111	5472	5833	6194	6538	6916
L327	346	437	798	1159	1520	1881	2242	2603	2964	3325	3686	4047	4408	4769	5130	5491	5852	6213	6574	6918
L328	347	456	817	1178	1539	1900	2261	2622	2983	3344	3705	4066	4427	4771	5149	5510	5871	6232	6593	6954
L329	348	475	836	1197	1558	1919	2280	2641	3002	3363	3707	4085	4446	4807	5168	5529	5890	6251	6612	6973
L330	349	494	855	1216	1577	1938	2299	2660	3021	3382	3743	4104	4448	4826	5187	5548	5909	6270	6631	6992
L331	350	513	874	1235	1596	1957	2318	2662	3040	3401	3762	4123	4484	4845	5206	5567	5928	6289	6650	7011
L332	351	532	893	1254	1615	1976	2337	2698	3059	3420	3781	4142	4503	4864	5225	5586	5947	6291	6669	7030
L333	352	551	912	1273	1634	1995	2356	2717	3078	3439	3800	4161	4522	4883	5244	5588	5966	6327	6688	7049
L334	353	570	931	1292	1653	1997	2375	2736	3097	3458	3819	4180	4541	4902	5263	5624	5985	6346	6707	7068
L335	354	589	950	1311	1672	2033	2394	2755	3116	3477	3838	4199	4560	4921	5282	5643	5987	6365	6726	7087
L336	355	608	969	1330	1691	2052	2396	2774	3135	3496	3857	4218	4579	4940	5301	5662	6023	6384	6745	7106
L337	356	627	988	1349	1693	2071	2432	2793	3154	3515	3876	4237	4598	4959	5320	5681	6042	6403	6764	7125
L338	357	646	1007	1368	1729	2090	2451	2812	3173	3534	3895	4256	4617	4978	5322	5700	6061	6422	6783	7144
L339	358	665	1026	1387	1748	2109	2470	2831	3192	3536	3914	4275	4636	4997	5358	5719	6080	6441	6802	7163
L340	359	684	1045	1406	1767	2128	2489	2850	3211	3572	3933	4277	4655	5016	5377	5738	6099	6460	6821	7182
L341	360	703	1064	1425	1786	2147	2508	2869	3213	3591	3952	4313	4674	5035	5396	5757	6118	6479	6840	7201
L342	361	722	1066	1444	1805	2166	2527	2888	3249	3610	3971	4332	4693	5054	5415	5776	6137	6498	6859	7220

L343	362	741	1102	1446	1824	2185	2546	2907	3268	3629	3990	4351	4712	5073	5434	5795	6156	6517	6878	7239
L344	363	400	761	1122	1483	1844	2205	2566	2927	3288	3649	4010	4371	4732	5093	5454	5815	6176	6537	6898
L345	364	419	780	1141	1502	1863	2224	2585	2946	3307	3668	4029	4390	4751	5112	5473	5834	6195	6556	6899
L346	365	438	799	1160	1521	1882	2243	2604	2965	3326	3669	4048	4409	4770	5131	5492	5853	6214	6575	6936
L347	366	457	818	1179	1540	1901	2262	2605	2984	3345	3706	4067	4428	4789	5150	5511	5872	6233	6594	6955
L348	367	476	837	1198	1559	1920	2281	2642	3003	3364	3725	4086	4447	4808	5169	5512	5891	6252	6613	6974
L349	368	495	856	1217	1578	1939	2300	2661	3022	3383	3744	4105	4466	4827	5188	5549	5892	6271	6632	6993
L350	369	514	875	1236	1579	1958	2319	2680	3041	3402	3763	4124	4485	4846	5207	5568	5929	6290	6651	7012
L351	370	533	894	1255	1616	1977	2338	2699	3060	3403	3782	4143	4504	4865	5226	5587	5948	6309	6670	7031
L352	371	552	913	1274	1635	1996	2357	2718	3061	3440	3801	4162	4523	4884	5245	5606	5967	6328	6689	7050
L353	372	571	932	1275	1654	2015	2376	2737	3098	3459	3820	4181	4542	4903	5264	5625	5986	6347	6708	7069
L354	373	590	951	1312	1673	2034	2395	2756	3117	3478	3839	4200	4561	4922	5283	5644	6005	6366	6709	7088
L355	374	609	970	1331	1692	2053	2414	2775	3136	3497	3858	4219	4580	4923	5302	5663	6024	6385	6746	7107
L356	375	628	989	1350	1711	2072	2433	2794	3155	3516	3877	4238	4581	4960	5321	5682	6043	6404	6765	7126
L357	376	647	1008	1369	1730	2091	2452	2813	3174	3535	3896	4257	4618	4979	5340	5701	6062	6405	6784	7145
L358	377	666	1027	1388	1749	2092	2471	2832	3193	3554	3915	4276	4637	4998	5359	5720	6081	6442	6803	7164
L359	378	685	1046	1407	1768	2129	2472	2851	3212	3573	3934	4295	4656	5017	5378	5739	6100	6461	6822	7183
L360	379	704	1065	1426	1787	2148	2509	2870	3231	3592	3953	4314	4675	5036	5379	5758	6119	6480	6841	7202
L361	380	723	1084	1445	1806	2167	2528	2889	3250	3611	3972	4315	4694	5055	5416	5777	6138	6499	6860	7221
L362	381	742	1085	1464	1825	2186	2547	2908	3269	3630	3991	4352	4713	5074	5435	5796	6157	6518	6879	7240

The line

L ₃₆₂	381	742	6898	7222	1221	1621	2021	2421	2821	3221	3621	4040	4440
	4840	5240	5640	6040	6440	6840	7240						

this line cannot intersect any line of A_{360} , then $A_{361} = A_{360} \cup L_{362} = \{ L_1, L_2, L_3, \dots, L_{362} \}$

is a (362, L)-spread

which is the maximum (H, ℓ)-spread of PG(3,19) can be obtained. Thus A_{361} is called a spread of (362) lines of PG(3,19) which partitions PG(3,19); that is every point of PG(3,19) lies in exactly one line of A_{361} , and every two lines of A_{361} are disjoint, i.e.

$\{ L_1, L_2, \dots, L_{362} \} = PG(3,19)$.

3.4. Results and conclusions

From the above results, the number of the lines in the projective space PG(3,19) is 7240. we found that the number of the lines do not intersect with some of them are (362) lines, these lines contains the whole points of the projective space PG(3,19), and called him, a (362, L)-spread, i.e. (362, L)-spread = $\{ L_1, L_2, \dots, L_{362} \} = PG(3,19) = \{ 1, 2, 3, \dots, 7240 \}$.

Moreover, we found that a (362, L)-spread is maximum complete (k, L)-spread in PG(3,19).

Table 2: is the points and lines for PG(3,19)

L1	2	21	40	59	78	97	116	135	154	173	192	211
	230	249	268	287	306	325	344	363	382	401	420	439
	458	477	496	515	534	553	572	591	610	629	648	667
	686	705	724	743	762	781	800	819	838	857	876	895
	914	933	952	971	990	1009	1028	1047	1066	1085	1104	1123
	1142	1161	1180	1199	1218	1237	1256	1275	1294	1313	1332	1351
	1370	1389	1408	1427	1446	1465	1484	1503	1522	1541	1560	1579
	1598	1617	1636	1655	1674	1693	1712	1731	1750	1769	1788	1807
	1826	1845	1864	1883	1902	1921	1940	1959	1978	1997	2016	2035
	2054	2073	2092	2111	2130	2149	2168	2187	2206	2225	2244	2263
	2282	2301	2320	2339	2358	2377	2396	2415	2434	2453	2472	2491

	2510	2529	2548	2567	2586	2605	2624	2643	2662	2681	2700	2719
	2738	2757	2776	2795	2814	2833	2852	2871	2890	2909	2928	2947
	2966	2985	3004	3023	3042	3061	3080	3099	3118	3137	3156	3175
	3194	3213	3232	3251	3270	3289	3308	3327	3346	3365	3384	3403
	3422	3441	3460	3479	3498	3517	3536	3555	3574	3593	3612	3631
	3650	3669	3688	3707	3726	3745	3764	3783	3802	3821	3840	3859
	3878	3897	3916	3935	3954	3973	3992	4011	4030	4049	4068	4087
	4106	4125	4144	4163	4182	4201	4220	4239	4258	4277	4296	4315
	4334	4353	4372	4391	4410	4429	4448	4467	4486	4505	4524	4543
	4562	4581	4600	4619	4638	4657	4676	4695	4714	4733	4752	4771
	4790	4809	4828	4847	4866	4885	4904	4923	4942	4961	4980	4999
	5018	5037	5056	5075	5094	5113	5132	5151	5170	5189	5208	5227
	5246	5265	5284	5303	5322	5341	5360	5379	5398	5417	5436	5455
	5474	5493	5512	5531	5550	5569	5588	5607	5626	5645	5664	5683
	5702	5721	5740	5759	5778	5797	5816	5835	5854	5873	5892	5911
	5930	5949	5968	5987	6006	6025	6044	6063	6082	6101	6120	6139
	6158	6177	6196	6215	6234	6253	6272	6291	6310	6329	6348	6367
	6386	6405	6424	6443	6462	6481	6500	6519	6538	6557	6576	6595
	6614	6633	6652	6671	6690	6709	6728	6747	6766	6785	6804	6823
	6842	6861	6880	6899	6918	6937	6956	6975	6994	7013	7032	7051
		7070	7089	7108	7127	7146	7165	7184	7203	7222		
L2	1	21	22	23	24	25	26	27	28	29	30	31
	32	33	34	35	36	37	38	39	382	383	384	385
	386	387	388	389	390	391	392	393	394	395	396	397
	398	399	400	743	744	745	746	747	748	749	750	751
	752	753	754	755	756	757	758	759	760	761	1104	1105
	1106	1107	1108	1109	1110	1111	1112	1113	1114	1115	1116	1117
	1118	1119	1120	1121	1122	1465	1466	1467	1468	1469	1470	1471
	1472	1473	1474	1475	1476	1477	1478	1479	1480	1481	1482	1483
	1826	1827	1828	1829	1830	1831	1832	1833	1834	1835	1836	1837
	1838	1839	1840	1841	1842	1843	1844	2187	2188	2189	2190	2191
	2192	2193	2194	2195	2196	2197	2198	2199	2200	2201	2202	2203
	2204	2205	2548	2549	2550	2551	2552	2553	2554	2555	2556	2557
	2558	2559	2560	2561	2562	2563	2564	2565	2566	2909	2910	2911
	2912	2913	2914	2915	2916	2917	2918	2919	2920	2921	2922	2923
	2924	2925	2926	2927	3270	3271	3272	3273	3274	3275	3276	3277
	3278	3279	3280	3281	3282	3283	3284	3285	3286	3287	3288	3631
	3632	3633	3634	3635	3636	3637	3638	3639	3640	3641	3642	3643
	3644	3645	3646	3647	3648	3649	3992	3993	3994	3995	3996	3997
	3998	3999	4000	4001	4002	4003	4004	4005	4006	4007	4008	4009
	4010	4353	4354	4355	4356	4357	4358	4359	4360	4361	4362	4363
	4364	4365	4366	4367	4368	4369	4370	4371	4714	4715	4716	4717
	4718	4719	4720	4721	4722	4723	4724	4725	4726	4727	4728	4729
	4730	4731	4732	5075	5076	5077	5078	5079	5080	5081	5082	5083
	5084	5085	5086	5087	5088	5089	5090	5091	5092	5093	5436	5437
	5438	5439	5440	5441	5442	5443	5444	5445	5446	5447	5448	5449
	5450	5451	5452	5453	5454	5797	5798	5799	5800	5801	5802	5803
	5804	5805	5806	5807	5808	5809	5810	5811	5812	5813	5814	5815
	6158	6159	6160	6161	6162	6163	6164	6165	6166	6167	6168	6169
	6170	6171	6172	6173	6174	6175	6176	6519	6520	6521	6522	6523
	6524	6525	6526	6527	6528	6529	6530	6531	6532	6533	6534	6535
	6536	6537	6880	6881	6882	6883	6884	6885	6886	6887	6888	6889
		6890	6891	6892	6893	6894	6895	6896	6897	6898		

L3	20	21	58	76	94	112	130	148	166	184	202	220
	238	256	274	292	310	328	346	364	382	419	437	455
	473	491	509	527	545	563	581	599	617	635	653	671
	689	707	725	743	780	798	816	834	852	870	888	906
	924	942	960	978	996	1014	1032	1050	1068	1086	1104	1141
	1159	1177	1195	1213	1231	1249	1267	1285	1303	1321	1339	1357
	1375	1393	1411	1429	1447	1465	1502	1520	1538	1556	1574	1592
	1610	1628	1646	1664	1682	1700	1718	1736	1754	1772	1790	1808
	1826	1863	1881	1899	1917	1935	1953	1971	1989	2007	2025	2043
	2061	2079	2097	2115	2133	2151	2169	2187	2224	2242	2260	2278
	2296	2314	2332	2350	2368	2386	2404	2422	2440	2458	2476	2494
	2512	2530	2548	2585	2603	2621	2639	2657	2675	2693	2711	2729
	2747	2765	2783	2801	2819	2837	2855	2873	2891	2909	2946	2964
	2982	3000	3018	3036	3054	3072	3090	3108	3126	3144	3162	3180
	3198	3216	3234	3252	3270	3307	3325	3343	3361	3379	3397	3415
	3433	3451	3469	3487	3505	3523	3541	3559	3577	3595	3613	3631
	3668	3686	3704	3722	3740	3758	3776	3794	3812	3830	3848	3866
	3884	3902	3920	3938	3956	3974	3992	4029	4047	4065	4083	4101
	4119	4137	4155	4173	4191	4209	4227	4245	4263	4281	4299	4317
	4335	4353	4390	4408	4426	4444	4462	4480	4498	4516	4534	4552
	4570	4588	4606	4624	4642	4660	4678	4696	4714	4751	4769	4787
	4805	4823	4841	4859	4877	4895	4913	4931	4949	4967	4985	5003
	5021	5039	5057	5075	5112	5130	5148	5166	5184	5202	5220	5238
	5256	5274	5292	5310	5328	5346	5364	5382	5400	5418	5436	5473
	5491	5509	5527	5545	5563	5581	5599	5617	5635	5653	5671	5689
	5707	5725	5743	5761	5779	5797	5834	5852	5870	5888	5906	5924
	5942	5960	5978	5996	6014	6032	6050	6068	6086	6104	6122	6140
	6158	6195	6213	6231	6249	6267	6285	6303	6321	6339	6357	6375
	6393	6411	6429	6447	6465	6483	6501	6519	6556	6574	6592	6610
	6628	6646	6664	6682	6700	6718	6736	6754	6772	6790	6808	6826
	6844	6862	6880	6917	6935	6953	6971	6989	7007	7025	7043	7061
		7079	7097	7115	7133	7151	7169	7187	7205	7223		
L7240	20	39	57	75	93	111	129	147	165	183	201	
	219	237	255	273	291	309	327	345	363	383	401	438
	456	474	492	510	528	546	564	582	600	618	636	654
	672	690	708	726	743	780	798	816	834	852	870	888
	906	924	942	960	978	996	1014	1032	1050	1068	1086	1122
	1140	1158	1176	1194	1212	1230	1248	1266	1284	1302	1320	1338
	1356	1374	1392	1410	1428	1446	1482	1500	1518	1536	1554	1572
	1590	1608	1626	1644	1662	1680	1698	1716	1734	1752	1770	1788
	1825	1842	1860	1878	1896	1914	1932	1950	1968	1986	2004	2022
	2040	2058	2076	2094	2112	2130	2167	2185	2202	2220	2238	2256
	2274	2292	2310	2328	2346	2364	2382	2400	2418	2436	2454	2472
	2509	2527	2545	2562	2580	2598	2616	2634	2652	2670	2688	2706
	2724	2742	2760	2778	2796	2814	2851	2869	2887	2905	2922	2940
	2958	2976	2994	3012	3030	3048	3066	3084	3102	3120	3138	3156
	3193	3211	3229	3247	3265	3282	3300	3318	3336	3354	3372	3390
	3408	3426	3444	3462	3480	3498	3535	3553	3571	3589	3607	3625
	3642	3660	3678	3696	3714	3732	3750	3768	3786	3804	3822	3840
	3877	3895	3913	3931	3949	3967	3985	4002	4020	4038	4056	4074
	4092	4110	4128	4146	4164	4182	4219	4237	4255	4273	4291	4309
	4327	4345	4362	4380	4398	4416	4434	4452	4470	4488	4506	4524
	4561	4579	4597	4615	4633	4651	4669	4687	4705	4722	4740	4758
	4776	4794	4812	4830	4848	4866	4903	4921	4939	4957	4975	4993

5011	5029	5047	5065	5082	5100	5118	5136	5154	5172	5190	5208
5245	5263	5281	5299	5317	5335	5353	5371	5389	5407	5425	5442
5460	5478	5496	5514	5532	5550	5587	5605	5623	5641	5659	5677
5695	5713	5731	5749	5767	5785	5802	5820	5838	5856	5874	5892
5929	5947	5965	5983	6001	6019	6037	6055	6073	6091	6109	6127
6145	6162	6180	6198	6216	6234	6271	6289	6307	6325	6343	6361
6379	6397	6415	6433	6451	6469	6487	6505	6522	6540	6558	6576
6613	6631	6649	6667	6685	6703	6721	6739	6757	6775	6793	6811
6829	6847	6865	6882	6900	6918	6955	6973	6991	7009	7027	7045
	7063	7081	7099	7117	7135	7153	7171	7189	7207	7225	

4. Conclusion

In this study, we constructed and analyzed (k, L) –spreads in the projective space $PG(3,19)$. Beginning with two disjoint lines, we systematically extended the sets by adding lines that are mutually skew, forming larger (k, L) –spreads at each step. This method demonstrates the existence of configurations where lines partition the space into disjoint subspaces, revealing both the geometric regularity and algebraic symmetry inherent in finite projective 3-spaces over $GF(19)$

The results confirm that such structured spreads can be effectively generated using an iterative approach, providing insight into the combinatorial properties of projective geometries. These findings may serve as a foundation for further research on finite geometries, coding theory, and related applications in discrete mathematics.

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الانتشارات (K, L) في الفضاء الإسقاطي ثلاثي الأبعاد PG(3, q) على الحقل الجاليواسي GF(19)

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معلومات البحث	الملخص
الاستلام 08 تشرين ثاني 2025 المراجعة 14 كانون أول 2025 القبول 16 كانون أول 2025 النشر 31 كانون أول 2025	يهدف هذا البحث إلى دراسة الفضاء الإسقاطي ثلاثي الأبعاد PG(3,19) باستخدام المعادلات الجبرية. تم تحديد النقاط، والخطوط، والمستويات في هذا الفضاء بالإضافة إلى ذلك، تم بناء انتشار من النوع (K,L)، يُعرّف على أنه مجموعة من K من الخطوط المتباينة في PG(3,19) وقد تم إثبات أن أقصى انتشار كامل من النوع (K,L) في PG(3,19) يتكوّن من 362 خطاً، وأن هذا الانتشار يغطي جميع نقاط الفضاء، وبالتالي يشكل امتداداً كاملاً له.
الكلمات المفتاحية	
الفضاء الإسقاطي، PG(3,19)، الحقل الجاليواسي، الانتشار (K,L)، الخطوط المتباينة، الهندسة ثلاثية الأبعاد، الهندسة المنتهية، البنى التوافقية..	

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