

Decomposition matrices and Brauer trees of spin characters for S_{25} , $p=17$

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ABSTRACT

In this paper, we will find the Brauer trees and the decomposition matrices of the symmetric groups S_n , when $n=25$ for $p=17$ by using (r, r') -inducing for calculates modular characters from the ordinary characters of the symmetric group. Also, we find all the blocks of defect 0 and 1 and we will depend on the results of the decomposition matrices of the symmetric groups S_n when $n=24$ for $p=17$.

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1. Introduction

The characters of the covering group of the symmetric group is said to be spin characters and the relation between the projective (ordinary) and the modular characters is called decomposition matrix. We want to calculate the modular characters from the ordinary characters of the symmetric group S_n .

The graph connection of the irreducible ordinary characters in the block of defect one is called Brauer trees [1]. We will calculate the Brauer trees by use the decomposition matrices for the symmetric group.

Finally, we will find the Brauer trees and decomposition matrices for the symmetric group S_{25} mod 17 by dependence on [A. A. Yaseen, M. M. Jawad] [2].

By using [3], the degree of spin characters $\langle \alpha \rangle$, $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ is :

$$\deg \langle \alpha \rangle = 2^{\frac{n-m}{2}} \frac{n!}{\prod_{i=1}^m \alpha_i!} \prod_{1 \leq i < j \leq m} \frac{(\alpha_i - \alpha_j)}{(\alpha_i + \alpha_j)} \quad (1)$$

2. Brauer trees of spin characters for S_{25}

The Symmetric group S_{25} has 214 irreducible spin characters and 205 of $(17, \alpha)$ -regular classes, Then the decomposition matrix of the spin characters for S_{25} , $p=17$ has 214 rows and 205 columns [4].

By using [3], there are 139 blocks of S_{25} six of them B_1, B_2, B_3, B_4, B_5 and B_6 of defect 1 and all the 139 remaining characters form their own blocks B_7, \dots, B_{139} of defect zero.

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By using [5], we get in the principal block B_1 , all the irreducible modular spin characters (i.m.s.) of the decomposition matrix are double and $\langle \beta \rangle \neq \langle \beta' \rangle$. The principle block B_1 contains the irreducible spin characters $\{ \langle 25 \rangle^*, \langle 17,8 \rangle, \langle 17,8' \rangle, \langle 16,8,1 \rangle^*, \langle 15,8,2 \rangle^*, \langle 14,8,3 \rangle^*, \langle 13,8,4 \rangle^*, \langle 12,8,5 \rangle^*, \langle 11,8,6 \rangle^*, \langle 10,8,7 \rangle^* \}$ with 17-bar core $\langle 8 \rangle$.

By using [5], we get in the principal block B_2 , all the irreducible modular spin characters (i.m.s.) of the decomposition matrix are double and $\langle \beta \rangle \neq \langle \beta' \rangle$. The principle block B_2 contains the irreducible spin characters $\{ \langle 22,2,1 \rangle^*, \langle 19,5,1 \rangle^*, \langle 18,5,2 \rangle^*, \langle 17,5,2,1 \rangle, \langle 17,5,2,1' \rangle, \langle 14,5,3,2,1 \rangle^*, \langle 13,5,4,2,1 \rangle^*, \langle 11,6,5,2,1 \rangle^*, \langle 10,7,5,2,1 \rangle^*, \langle 9,8,5,2,1 \rangle^* \}$ with 17-bar core $\langle 5,2,1 \rangle$.

By using [5], we get in the principal block B_3 , all the irreducible modular spin characters (i.m.s.) of the decomposition matrix are double and $\langle \beta \rangle \neq \langle \beta' \rangle$. The principle block B_3 contains the irreducible spin characters $\{ \langle 21,3,1 \rangle^*, \langle 20,4,1 \rangle^*, \langle 18,4,3 \rangle^*, \langle 17,4,3,1 \rangle, \langle 17,4,3,1' \rangle, \langle 15,4,3,2,1 \rangle^*, \langle 12,5,4,3,1 \rangle^*, \langle 11,6,4,3,1 \rangle^*, \langle 10,7,4,3,1 \rangle^*, \langle 9,8,4,3,1 \rangle^* \}$ with 17-bar core $\langle 4,3,1 \rangle$.

By using [5], we get in the block B_4 , all the irreducible modular spin characters (i.m.s.) of the decomposition matrix are associate and $\langle \beta \rangle \neq \langle \beta' \rangle$. The block B_4 contains the irreducible spin characters $\{ \langle 24,1 \rangle, \langle 24,1' \rangle, \langle 18,7 \rangle, \langle 18,7' \rangle, \langle 17,7,1 \rangle^*, \langle 15,7,2,1 \rangle, \langle 15,7,2,1' \rangle, \langle 14,7,3,1 \rangle, \langle 14,7,3,1' \rangle, \langle 13,7,4,1 \rangle, \langle 13,7,4,1' \rangle, \langle 12,7,5,1 \rangle, \langle 12,7,5,1' \rangle, \langle 11,7,6,1 \rangle, \langle 11,7,6,1' \rangle, \langle 9,8,7,1 \rangle, \langle 9,8,7,1' \rangle \}$ has 17-bar core $\langle 7,1 \rangle$.

By using [5], we get in the block B_5 , all the irreducible modular spin characters (i.m.s.) of the decomposition matrix are associate and $\langle \beta \rangle \neq \langle \beta' \rangle$. The block B_5 , contains the irreducible spin characters $\{ \langle 23,2 \rangle, \langle 23,2' \rangle, \langle 19,6 \rangle, \langle 19,6' \rangle, \langle 17,6,2 \rangle^*, \langle 16,6,2,1 \rangle, \langle 16,6,2,1' \rangle, \langle 14,6,3,2 \rangle, \langle 14,6,3,2' \rangle, \langle 13,6,4,2 \rangle, \langle 13,6,4,2' \rangle, \langle 12,6,5,2 \rangle, \langle 12,6,5,2' \rangle, \langle 10,7,6,2 \rangle, \langle 10,7,6,2' \rangle, \langle 9,8,6,2 \rangle, \langle 9,8,6,2' \rangle \}$ has 17-bar core $\langle 6,2 \rangle$.

By using [5], we get in the block B_6 , all the irreducible modular spin characters (i.m.s.) of the decomposition matrix are associate and $\langle \beta \rangle \neq \langle \beta' \rangle$. The block B_6 , contains the irreducible spin characters $\{ \langle 22,3 \rangle, \langle 22,3' \rangle, \langle 20,5 \rangle, \langle 20,5' \rangle, \langle 17,5,3 \rangle^*, \langle 16,5,3,1 \rangle, \langle 16,5,3,1' \rangle, \langle 15,5,3,2 \rangle, \langle 15,5,3,2' \rangle, \langle 13,5,4,3 \rangle, \langle 13,5,4,3' \rangle, \langle 11,6,5,3 \rangle, \langle 11,6,5,3' \rangle, \langle 10,7,5,3 \rangle, \langle 10,7,5,3' \rangle, \langle 9,8,5,3 \rangle, \langle 9,8,5,3' \rangle \}$ has 17-bar core $\langle 5,3 \rangle$.

Lemma (2.1.):

The Brauer tree for the block B_1 is:

$$\langle 25 \rangle^* - \langle 17,8 \rangle = \langle 17,8' \rangle - \langle 16,8,1 \rangle^* - \langle 15,8,2 \rangle^* - \langle 14,8,3 \rangle^* - \langle 13,8,4 \rangle^* - \langle 12,8,5 \rangle^* - \langle 11,8,6 \rangle^* - \langle 10,8,7 \rangle^*$$

Proof :

By using [3,6], we get :

$$\deg \{ \langle 25 \rangle^*, \langle 16,8,1 \rangle^*, \langle 14,8,3 \rangle^*, \langle 12,8,5 \rangle^*, \langle 10,8,7 \rangle^* \} \equiv 16 \pmod{17}$$

$$\deg \{ \langle 17,8 \rangle = \langle 17,8' \rangle, \langle 15,8,2 \rangle^*, \langle 13,8,4 \rangle^*, \langle 11,8,6 \rangle^* \} \equiv -16 \pmod{17}$$

By using (r, r') - inducing of the principal irreducible spin characters (p.i.s.) of S_{24} to S_{25} we have:

$$d_{41} \uparrow^{(8,10)} S_{25} = D_1$$

$$d_{43} \uparrow^{(8,10)} S_{25} = D_2$$

$$d_{45} \uparrow^{(8,10)} S_{25} = D_3$$

$$d_{47} \uparrow^{(8,10)} S_{25} = D_4$$

$$d_{49} \uparrow^{(8,10)} S_{25} = D_5$$

$$d_{51} \uparrow^{(8,10)} S_{25} = D_6$$

$$d_{53} \uparrow^{(8,10)} S_{25} = D_7$$

$$d_{55} \uparrow^{(8,10)} S_{24} = D_8$$

Table 1 : The decomposition matrix for this block $D_{25,17}^{(1)}$

The spin characters		The decomposition matrix of B_1							
$\langle 25 \rangle^*$	1								
$\langle 17,8 \rangle$	1	1							
$\langle 17,8 \rangle^*$	1	1							
$\langle 16,8,1 \rangle^*$		1	1						
$\langle 15,8,2 \rangle^*$			1	1					
$\langle 14,8,3 \rangle^*$				1	1				
$\langle 13,8,4 \rangle^*$					1	1			
$\langle 12,8,5 \rangle^*$						1	1		
$\langle 11,8,6 \rangle^*$							1	1	1
$\langle 10,8,7 \rangle^*$								1	1
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9

Lemma (2.2.):

The Brauer tree for the block B_2 is:

$$\langle 22,2,1 \rangle^* - \langle 19,5,1 \rangle^* - \langle 18,5,2 \rangle^* - \langle 17,5,2,1 \rangle = \langle 17,5,2,1 \rangle^* - \langle 14,5,3,2,1 \rangle^* - \langle 13,5,4,2,1 \rangle^* - \langle 11,6,5,2,1 \rangle^* - \langle 10,7,5,2,1 \rangle^* - \langle 9,8,5,2,1 \rangle^*$$

Proof :

By using [3,6], we get :

$$\deg\{ \langle 22,2,1 \rangle^*, \langle 18,5,2 \rangle^*, \langle 14,5,3,2,1 \rangle^*, \langle 11,6,5,2,1 \rangle^*, \langle 9,8,5,2,1 \rangle^* \} \equiv 9 \pmod{17};$$

$$\deg\{ \langle 19,5,1 \rangle^*, \langle 17,5,2,1 \rangle = \langle 17,5,2,1 \rangle^*, \langle 13,5,4,2,1 \rangle^*, \langle 10,7,5,2,1 \rangle^* \} \equiv -9 \pmod{17}$$

By using (r, r') - inducing of the principal irreducible spin characters (p.i.s.) of S_{24} to S_{25} we have:

$$d_{25} \uparrow^{(5,13)} S_{25} = D_9$$

$$d_{27} \uparrow^{(5,13)} S_{25} = D_{10}$$

$$d_{29} \uparrow^{(5,13)} S_{25} = D_{11}$$

$$d_{31} \uparrow^{(5,13)} S_{25} = D_{12}$$

$$d_{33} \uparrow^{(5,13)} S_{25} = D_{13}$$

$$d_{35} \uparrow^{(5,13)} S_{25} = D_{14}$$

$$d_{37} \uparrow^{(5,13)} S_{25} = D_{15}$$

$$d_{39} \uparrow^{(5,13)} S_{25} = D_{16}$$

Table 2: The decomposition matrix for this block $D_{25,17}^{(2)}$

The spin characters		The decomposition matrix of B_2							
$\langle 22,2,1 \rangle^*$	1								
$\langle 19,5,1 \rangle^*$	1	1							
$\langle 17,5,2,1 \rangle^*$		1	1						
$\langle 17,5,2,1 \rangle$			1						
$\langle 17,5,2 \rangle^*$			1	1					
$\langle 14,5,3,2,1 \rangle^*$				1	1				
$\langle 13,5,4,2,1 \rangle^*$				1	1	1			
$\langle 11,6,5,2,1 \rangle^*$						1	1		
$\langle 10,7,5,2,1 \rangle^*$							1	1	1
$\langle 9,8,5,2,1 \rangle^*$								1	1
	D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}	D_{16}	D_{17}

Lemma (2.3.):

The Brauer tree for the block B_3 is:

$$\langle 21,3,1 \rangle^* - \langle 20,4,1 \rangle^* - \langle 18,4,3 \rangle^* - \langle 17,4,3,1 \rangle = \langle 17,4,3,1 \rangle' - \langle 15,4,3,2,1 \rangle^* - \langle 12,5,4,3,1 \rangle^* - \langle 11,6,4,3,1 \rangle^* - \langle 10,7,4,3,1 \rangle^* - \langle 9,8,4,3,1 \rangle^*$$

Proof :

By using [3,6], we get :

$$\deg\{ \langle 21,3,1 \rangle^*, \langle 18,4,3 \rangle^*, \langle 15,4,3,2,1 \rangle^*, \langle 11,6,4,3,1 \rangle^*, \langle 9,8,4,3,1 \rangle^* \} \equiv 11 \pmod{17};$$

$$\deg\{ \langle 20,4,1 \rangle^*, \langle 17,4,3,1 \rangle = \langle 17,4,3,1 \rangle', \langle 12,5,4,3,1 \rangle^*, \langle 10,7,4,3,1 \rangle^* \} \equiv -11 \pmod{17}$$

By using (r, r') - inducing of the principal irreducible spin characters (p.i.s.) of S_{24} to S_{25} we have:

$$d_{17} \uparrow^{(0,1)} S_{24} = D_{17}$$

$$d_{27} \uparrow^{(3,15)} S_{24} = D_{18}$$

$$d_{29} \uparrow^{(3,15)} S_{24} = D_{19}$$

$$d_{31} \uparrow^{(3,15)} S_{24} = D_{20}$$

$$d_{33} \uparrow^{(3,15)} S_{24} = D_{21}$$

$$d_{35} \uparrow^{(3,15)} S_{24} = D_{22}$$

$$d_{37} \uparrow^{(3,15)} S_{24} = D_{23}$$

$$d_{39} \uparrow^{(3,15)} S_{24} = D_{24}$$

Table 3: The decomposition matrix for this block $D_{25,17}^{(3)}$

The spin characters	The decomposition matrix of B_3							
$\langle 21,3,1 \rangle^*$	1							
$\langle 20,4,1 \rangle^*$	1	1						
$\langle 18,4,3 \rangle^*$		1	1					
$\langle 17,4,3,1 \rangle$			1					
$\langle 17,4,3,1 \rangle'$			1	1				
$\langle 15,4,3,2,1 \rangle^*$				1	1			
$\langle 12,5,4,3,1 \rangle^*$				1	1	1		
$\langle 11,6,4,3,1 \rangle^*$						1	1	
$\langle 10,7,4,3,1 \rangle^*$							1	1
$\langle 9,8,4,3,1 \rangle^*$								1
	D_{17}	D_{18}	D_{19}	D_{20}	D_{21}	D_{22}	D_{23}	D_{24}

Lemma (2.4.):

The Brauer tree for the block B_4 is:

$$\begin{array}{c} \langle 24,1 \rangle - \langle 18,7 \rangle \searrow \nearrow \langle 15,7,2,1 \rangle - \langle 14,7,3,1 \rangle - \langle 13,7,4,1 \rangle - \langle 12,7,5,1 \rangle - \langle 11,7,6,1 \rangle - \langle 9,8,7,1 \rangle \\ \qquad \qquad \qquad \nearrow \searrow \langle 17,7,1 \rangle^* \\ \langle 24,1 \rangle' - \langle 18,7 \rangle' \nearrow \searrow \langle 15,7,2,1 \rangle' - \langle 14,7,3,1 \rangle' - \langle 13,7,4,1 \rangle' - \langle 12,7,5,1 \rangle' - \langle 11,7,6,1 \rangle' - \langle 9,8,7,1 \rangle' \end{array}$$

Proof :

By using [3,6], we get :

$$\deg\{ \langle 24,1 \rangle, \langle 24,1 \rangle', \langle 17,7,1 \rangle^*, \langle 14,7,3,1 \rangle, \langle 14,7,3,1 \rangle', \langle 12,7,5,1 \rangle, \langle 12,7,5,1 \rangle', \langle 9,8,7,1 \rangle, \langle 9,8,7,1 \rangle' \} \equiv 14 \pmod{17};$$

$$\deg\{ \langle 18,7 \rangle, \langle 18,7 \rangle', \langle 15,7,2,1 \rangle, \langle 15,7,2,1 \rangle', \langle 13,7,4,1 \rangle, \langle 13,7,4,1 \rangle', \langle 11,7,6,1 \rangle, \langle 11,7,6,1 \rangle' \} \equiv -14 \pmod{17}$$

By using (r, r') - inducing of the principal irreducible spin characters (p.i.s.) of S_{24} to S_{25} we have:

$$d_1 \uparrow^{(7,11)} S_{25} = K_1 = D_{25} + D_{26}$$

$$d_2 \uparrow^{(7,11)} S_{25} = K_2 = D_{27} + D_{28}$$

$$d_{45} \uparrow^{(0,1)} S_{25} = D_{29}, \quad d_{46} \uparrow^{(0,1)} S_{25} = D_{30}$$

$$d_{47} \uparrow^{(0,1)} S_{25} = D_{31}, \quad d_{48} \uparrow^{(0,1)} S_{25} = D_{32}$$

$$d_{49} \uparrow^{(0,1)} S_{25} = D_{33}, \quad d_{50} \uparrow^{(0,1)} S_{25} = D_{34}$$

$$\begin{aligned} d_{51}\uparrow^{(0,1)}S_{25} &= D_{35} & , & & d_{52}\uparrow^{(0,1)}S_{25} &= D_{36} \\ d_{53}\uparrow^{(0,1)}S_{25} &= D_{37} & , & & d_{54}\uparrow^{(0,1)}S_{25} &= D_{38} \\ d_{55}\uparrow^{(0,1)}S_{25} &= D_{39} & , & & d_{56}\uparrow^{(0,1)}S_{25} &= D_{40} \end{aligned}$$

Since $\langle 24,1 \rangle \neq \langle 24,1 \rangle'$ on $(17, \alpha)$ -regular classes then K_1 is split to D_{25} and D_{26}

$\langle 18,7,1 \rangle \downarrow_{(0,1)} S_{25} = \langle 18,7 \rangle + \langle 17,7,1 \rangle^* = D_{27}$. Since $\langle 18,7,1 \rangle$ i.m.s in S_{26} .

$\langle 18,7,1 \rangle' \downarrow_{(0,1)} S_{25} = \langle 18,7 \rangle' + \langle 17,7,1 \rangle^* = D_{28}$. Since $\langle 18,7,1 \rangle'$ i.m.s in S_{26} .

Then K_2 split to D_{27} and D_{28} .

Table 4 : The decomposition matrix for this block $D_{25,17}^{(4)}$

The spin characters	The decomposition matrix of B_4															
$\langle 24,1 \rangle$																
$\langle 24,1 \rangle'$																
$\langle 18,7 \rangle$																
$\langle 18,7 \rangle'$																
$\langle 17,7,1 \rangle^*$																
$\langle 15,7,2,1 \rangle$																
$\langle 15,7,2,1 \rangle'$																
$\langle 14,7,3,1 \rangle$																
$\langle 14,7,3,1 \rangle'$																
$\langle 13,7,4,1 \rangle$																
$\langle 13,7,4,1 \rangle'$																
$\langle 12,7,5,1 \rangle$																
$\langle 12,7,5,1 \rangle'$																
$\langle 11,7,6,1 \rangle$																
$\langle 11,7,6,1 \rangle'$																
$\langle 9,8,7,1 \rangle$																
$\langle 9,8,7,1 \rangle'$																
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	3	6	7	8	9	6	1	2	2	4	2	6	7	8	9	6

Lemma (2.5.):

The Brauer tree for the block B_5 is:

$$\langle 23,2 \rangle - \langle 19,6 \rangle \setminus \quad / \quad \langle 16,6,2,1 \rangle - \langle 14,6,3,2 \rangle - \langle 13,6,4,2 \rangle - \langle 12,6,5,2 \rangle - \langle 10,7,6,2 \rangle - \langle 9,8,6,2 \rangle$$

$$\langle 23,2 \rangle' - \langle 19,6 \rangle' \setminus \quad / \quad \langle 17,6,2 \rangle^* \setminus \quad / \quad \langle 16,6,2,1 \rangle' - \langle 14,6,3,2 \rangle' - \langle 13,6,4,2 \rangle' - \langle 12,6,5,2 \rangle' - \langle 10,7,6,2 \rangle' - \langle 9,8,6,2 \rangle'$$

Proof :

By using [3,6], we get :

$\deg\{ \langle 23,2 \rangle, \langle 23,2 \rangle', \langle 17,6,2 \rangle^*, \langle 14,6,3,2 \rangle, \langle 14,6,3,2 \rangle', \langle 12,6,5,2 \rangle, \langle 12,6,5,2 \rangle', \langle 9,8,6,2 \rangle, \langle 9,8,6,2 \rangle' \} \equiv 10 \pmod{17};$

$\deg\{ \langle 19,6 \rangle, \langle 19,6 \rangle', \langle 16,6,2,1 \rangle, \langle 16,6,2,1 \rangle', \langle 13,6,4,2 \rangle, \langle 13,6,4,2 \rangle', \langle 10,7,6,2 \rangle, \langle 10,7,6,2 \rangle' \} \equiv -10 \pmod{17}$

By using (r, r') - inducing of the principal irreducible spin characters (p.i.s.) of S_{24} to S_{25} we have:

$$d_1 \uparrow^{(2,16)} S_{25} = K_3 = D_{41} + D_{42}$$

$$d_2 \uparrow^{(2,16)} S_{25} = K_4 = D_{43} + D_{44}$$

$$d_3 \uparrow^{(2,16)} S_{25} = K_5 = D_{45} + D_{46}$$

$$d_4 \uparrow^{(2,16)} S_{25} = K_6 = D_{47} + D_{48}$$

$$d_5 \uparrow^{(2,16)} S_{25} = K_7 = D_{49} + D_{50}$$

$$d_6 \uparrow^{(2,16)} S_{25} = K_8 = D_{51} + D_{52}$$

$$d_7 \uparrow^{(2,16)} S_{25} = K_9 = D_{53} + D_{54}$$

$$d_8 \uparrow^{(2,16)} S_{25} = K_{10} = D_{55} + D_{56}$$

Since $\langle 23,2 \rangle \neq \langle 23,2 \rangle'$, $\langle 19,6 \rangle \neq \langle 19,6 \rangle'$, $\langle 16,6,2,1 \rangle \neq \langle 16,6,2,1 \rangle'$, $\langle 14,6,3,2 \rangle \neq \langle 14,6,3,2 \rangle'$, $\langle 13,6,4,2 \rangle \neq \langle 13,6,4,2 \rangle'$, $\langle 12,6,5,2 \rangle \neq \langle 12,6,5,2 \rangle'$, $\langle 10,7,6,2 \rangle \neq \langle 10,7,6,2 \rangle'$, $\langle 9,8,6,2 \rangle \neq \langle 9,8,6,2 \rangle'$ on $(17, \alpha)$ -regular classes, then $K_3, K_4, K_5, K_6, K_7, K_8, K_9$ and K_{10} are split respectively.

Table 5: The decomposition matrix for this block $D_{25,17}^{(5)}$

The spin characters	The decomposition matrix of B_5															
$\langle 23,2 \rangle$																
$\langle 23,2 \rangle'$	1															
$\langle 19,6 \rangle$		1														
$\langle 19,6 \rangle'$	1		1													
$\langle 17,6,2 \rangle^*$			1	1	1	1										
$\langle 16,6,2,1 \rangle$					1		1									
$\langle 16,6,2,1 \rangle'$						1		1								
$\langle 14,6,3,2 \rangle$							1		1							
$\langle 14,6,3,2 \rangle'$								1		1						
$\langle 13,6,4,2 \rangle$									1		1					
$\langle 13,6,4,2 \rangle'$										1		1				
$\langle 12,6,5,2 \rangle$											1		1			
$\langle 12,6,5,2 \rangle'$												1		1		
$\langle 10,7,6,2 \rangle$													1		1	
$\langle 10,7,6,2 \rangle'$														1		1
$\langle 9,8,6,2 \rangle$															1	
$\langle 9,8,6,2 \rangle'$																1
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5
	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7

Lemma (2.6.):

The Brauer tree for the block B_6 is:

$$\langle 22,3 \rangle - \langle 20,5 \rangle \setminus \begin{array}{c} \nearrow \langle 16,5,3,1 \rangle - \langle 15,5,3,2 \rangle - \langle 13,5,4,3 \rangle - \langle 11,6,5,3 \rangle - \langle 10,7,5,3 \rangle - \langle 9,8,5,3 \rangle \\ \searrow \langle 17,5,3 \rangle^* \end{array}$$

$$\langle 22,3 \rangle' - \langle 20,5 \rangle' \setminus \begin{array}{c} \nearrow \langle 16,5,3,1 \rangle' - \langle 15,5,3,2 \rangle' - \langle 13,5,4,3 \rangle' - \langle 11,6,5,3 \rangle' - \langle 10,7,5,3 \rangle' - \langle 9,8,5,3 \rangle' \\ \searrow \end{array}$$

Proof :

By using [3,6], we get :

$$\deg\{ \langle 22,3 \rangle, \langle 22,3 \rangle', \langle 17,5,3 \rangle^*, \langle 15,5,3,2 \rangle, \langle 15,5,3,2 \rangle', \langle 11,6,5,3 \rangle, \langle 11,6,5,3 \rangle', \langle 9,8,5,3 \rangle, \langle 9,8,5,3 \rangle' \} \equiv 10 \pmod{17};$$

$\deg\{ \langle 20,5 \rangle, \langle 20,5' \rangle, \langle 16,5,3,1 \rangle, \langle 16,5,3,1' \rangle, \langle 13,5,4,3 \rangle, \langle 13,5,4,3' \rangle, \langle 10,7,5,3 \rangle, \langle 10,7,5,3' \rangle \} \equiv -10 \pmod{17}$

By using (r,r') - inducing of the principal irreducible spin characters (p.i.s.) of \mathcal{S}_{24} to \mathcal{S}_{25} we have:

$$d_9 \uparrow^{(3,15)} \mathcal{S}_{25} = K_{11} = D_{57} + D_{58}$$

$$d_{10} \uparrow^{(3,15)} \mathcal{S}_{25} = K_{12} = D_{59} + D_{60}$$

$$d_{11} \uparrow^{(3,15)} \mathcal{S}_{25} = K_{13} = D_{61} + D_{62}$$

$$d_{12} \uparrow^{(3,15)} \mathcal{S}_{25} = K_{14} = D_{63} + D_{64}$$

$$d_{13} \uparrow^{(3,15)} \mathcal{S}_{25} = K_{15} = D_{65} + D_{66}$$

$$d_{14} \uparrow^{(3,15)} \mathcal{S}_{25} = K_{16} = D_{67} + D_{68}$$

$$d_{15} \uparrow^{(3,15)} \mathcal{S}_{25} = K_{17} = D_{69} + D_{70}$$

$$d_{16} \uparrow^{(3,15)} \mathcal{S}_{25} = K_{18} = D_{71} + D_{72}$$

Since $\langle 22,3 \rangle \neq \langle 22,3' \rangle$, $\langle 20,5 \rangle \neq \langle 20,5' \rangle$, $\langle 16,5,3,1 \rangle \neq \langle 16,5,3,1' \rangle$, $\langle 15,5,3,2 \rangle \neq \langle 15,5,3,2' \rangle$, $\langle 13,5,4,3 \rangle \neq \langle 13,5,4,3' \rangle$, $\langle 11,6,5,3 \rangle \neq \langle 11,6,5,3' \rangle$, $\langle 10,7,6,2 \rangle \neq \langle 10,7,6,2' \rangle$, $\langle 9,8,5,3 \rangle \neq \langle 9,8,5,3' \rangle$ on $(17, \alpha)$ -regular classes, then K_{11} , K_{12} , K_{13} , K_{14} , K_{15} , K_{16} , K_{17} and K_{18} are split respectively.

Table 6 : The decomposition matrix for this block $\mathcal{D}_{25,17}^{(6)}$

The spin characters		The decomposition matrix of \mathcal{B}_6													
$\langle 22,3 \rangle$															
$\langle 22,3' \rangle$	1														
$\langle 20,5 \rangle$		1													
$\langle 20,5' \rangle$	1		1												
$\langle 17,5,3 \rangle^*$			1	1	1	1									
$\langle 16,5,3,1 \rangle$					1		1								
$\langle 16,5,3,1' \rangle$						1		1							
$\langle 15,5,3,2 \rangle$							1		1						
$\langle 15,5,3,2' \rangle$								1		1					
$\langle 13,5,4,3 \rangle$									1		1				
$\langle 13,5,4,3' \rangle$										1		1			
$\langle 11,6,5,3 \rangle$											1		1		
$\langle 11,6,5,3' \rangle$												1		1	
$\langle 10,7,5,3 \rangle$													1		1
$\langle 10,7,5,3' \rangle$														1	
$\langle 9,8,5,3 \rangle$															1
$\langle 9,8,5,3' \rangle$															
		I	I	I	I	I	I	I	I	I	I	I	I	I	I
		5	5	6	6	6	6	6	6	6	6	6	6	7	7
		8	9	0	1	2	3	4	5	6	7	8	9	0	1

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مصفوفات التجزئة وأشجار براور للمشخصات الاسقاطية ل S_{25} و $P=17$

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المعلومات البحث	الملخص
الاستلام القبول النشر	في هذا البحث سوف نقوم بإيجاد أشجار براور ومصفوفات التجزئة للزمر التناظرية S_n عندما $n=25$, $p=17$ باستخدام (r, r') المولدة لحساب المشخصات الاسقاطية من المشخصات العادية للزمر التناظرية كذلك سوف نقوم بإيجاد كل القطع من النوع 1 و 0 بالاعتماد على نتائج مصفوفات التجزئة للزمر التناظرية S_n عندما $n=24$ $p=17$.
الكلمات المفتاحية	
أشجار براور ، المشخصات الاسقاطية ، مصفوفات التجزئة	

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